The graph of $g$ is obtained by reflecting the graph of $f$ about the $y$-axis. For $h$, reflect the graph of $f$ about the line $y = x$. For $k$, reflect the graph of $f$ about the $x$-axis and then we shift 2 units upward.

$h = 2 - f(x)$
2) \( h(x) = 6x^4 - 2x^2 + 1 \)

We may take \( f(x) = 6x^2 - 2x + 1 \) and \( g(x) = x^2 \) and then write \( h(x) = f \circ g(x) \)

This is not the only acceptable answer. We may also consider \( f(x) = \frac{3}{2} x^2 - x + 1 \) and \( g(x) = 2x^2 \) so \( h(x) = f \circ g(x) \).

3) \(-f(0) = 1\) (\( f \) is defined in \( x = 0 \) and has value 1 for this \( x \))

\( \lim_{x \to 0} f(x) \) is not defined, because \( \lim_{x \to 0^+} f(x) \) and \( \lim_{x \to 0^-} f(x) \) exist but are different, as we shall see in a moment.

From the graph we can see that:
- \( \lim_{x \to 0^+} f(x) = 2 \); \( \lim_{x \to 0^-} f(x) = 1 \)

- Even if \( g(1) = -2 \) the limit \( \lim_{x \to 1} g(x) \) exists and is 0.

since \( \lim_{x \to 1^-} g(x) = \lim_{x \to 1^+} g(x) = 0 \)

- \( \lim (f(x) + g(x/2)) = \lim_{x \to -2} f(x) + \lim_{x \to -2} g(x/2) = 0 + (-2) = -2 \)

We used the fact that \( \lim_{x \to -2} \frac{x}{2} = -1 \)

- We must calculate \( \lim_{x \to 1} \frac{f(x)}{g(x)} \)

From the graph we deduce that \( \lim_{x \to 1} f(x) = 0 \) and \( \lim_{x \to 1} g(x) = 0 \) so we can't know directly the limit.
\[ f(x) = -2x + 2 \quad g(x) = x - 1 \quad \text{(only for } x \neq 1) \]

\[
\lim_{x \to 1} \frac{f(x)}{g(x)} = \lim_{x \to 1} \frac{-2x + 2}{x - 1} = \lim_{x \to 1} \frac{-2(x - 1)}{x - 1} = -2
\]

\[ \lim_{x \to 3} (2f(x) - f(3)) = \lim_{x \to 3} 2f(x) - \lim_{x \to 3} f(3) = 2 \lim_{x \to 3} f(x) - f(3) \]

\[ = 2(-2) - 1 = -4 + 1 = -3. \]

From the graph we can see that there are possible problems only in 0 and 3. So we must check the continuity only in these points. On all other points being points of continuity for \( f \).

We've seen that \( \lim_{x \to 0^+} f(x) = 2 \neq \lim_{x \to 0^-} f(x) = 1 \) and \( f(0) = 1 \)

Also \( \lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x) = -2 \) but \( f(3) = 1 \)

So we deduce that \( f \) is continuous from the left in 0 and \( f \) is not continuous in 3.
5) Remaining area = Area of the circle - Area of the triangle, because the triangle is contained in the circle.

The circle has radius $x$, so its area is $\pi x^2$.
The triangle is equilateral, having all the edges of equal length $x$. Let's calculate its area.

![Equilateral triangle diagram](image)

Applying Pythagoras's theorem we have:

$$h^2 + \left(\frac{x}{2}\right)^2 = x^2 \Rightarrow h = \frac{x\sqrt{3}}{2}$$

So the area of the triangle $\Delta = \frac{h \cdot x}{2} = \frac{x^2 \sqrt{3}}{4}$

So our area is $A = \pi x^2 - \frac{x^2 \sqrt{3}}{4} = x^2 \left(\pi - \frac{\sqrt{3}}{4}\right)$

5) $f(x) = \frac{1}{x+2}$, $g(x) = \frac{x-1}{x}$

The domains are:

$\text{Dom}(f) = \mathbb{R} \setminus \{-2\}$, $\text{Dom}(g) = \mathbb{R} \setminus \{0\}$

$$(\text{fof})(x) = \frac{1}{x+2} = \frac{x+2}{2x+5}$$

$\text{Dom}(\text{fof}) = \mathbb{R} \setminus \{-2, \frac{-5}{2}\}$

$$(\text{fog})(x) = \frac{1}{x-1} + 2 = \frac{x}{3x-1}$$

$\text{Dom}(\text{fog}) = \mathbb{R} \setminus \{-2, \frac{1}{3}\}$

$$(\text{gof})(x) = \frac{1}{x+2} - 1 = -x - 1$$

$\text{Dom}(\text{gof}) = \mathbb{R} \setminus \{0\}$

$$(\text{gog})(x) = \frac{x-1}{x} - 1 = \frac{1}{1-x}$$

$\text{Dom}(\text{gog}) = \mathbb{R} \setminus \{0, 1\}$$
a) \( \lim_{x \to 2} 3x^2 - x - 2 = 3 \cdot 2^2 - 2 - 2 = 8 \)

b) \( \lim_{x \to 2} \frac{x^2 + x - 6}{x-2} = \lim_{x \to 2} \frac{(x+3)(x-2)}{x-2} = \lim_{x \to 2} x+3 = 5 \)

c) \( \lim_{q \to 2} \frac{2q^2 + 5}{q+2} = \frac{2 \cdot 2^2 + 5}{2+2} = \frac{13}{4} \)

d) We have \( x^2 + 1 \geq f(x) \geq 4x - 3 \) for all \( x \).

\( \Rightarrow \lim_{x \to 2} (x^2 + 1) \geq \lim_{x \to 2} f(x) \geq \lim_{x \to 2} (4x - 3) \)

But \( \lim_{x \to 2} (x^2 + 1) = 5 \) and \( \lim_{x \to 2} 4x - 3 = 5 \).

Applying The Squeeze Theorem we have \( \lim_{x \to 2} f(x) = 5 \).

e) \( \lim_{y \to -3} |y+3| = \lim_{y \to -3} 0 \)

f) \( \lim_{h \to 0} \frac{(2+h)^2 - 2}{h} = \lim_{h \to 0} \frac{4 + 4h + h^2 - 2}{h} = \lim_{h \to 0} \frac{h^2 + 4h - 2}{h} \)

We have \( \lim_{h \to 0} h^2 + 4h - 2 = -2 \) and \( \lim_{h \to 0} h = 0 \)

Since \( h \) could be positive or negative the limit is not defined.
Of course, what I really meant to ask was the following:

\[ \lim_{h \to 2} \frac{(h + 2)^2 - 4}{h} \]

To do this, we first try plugging in, and see that we get \( \frac{0}{0} \), so its time to do more work. Notice that after squaring out \((h + 2)^2\), we get

\[
\lim_{h \to 2} \frac{(h^2 + 4h + 4) - 4}{h} = \lim_{h \to 2} \frac{h^2 + 4h}{h} \\
= \lim_{h \to 2} \frac{h(h + 4)}{h} \\
= \lim_{h \to 2}(h + 4) = 2 + 4 = 6
\]
\[ 8^{\frac{-1000}{\sqrt{2^{10000}}} - \frac{16000}{16^{500}}} = \left(2^{4}\right)^{\frac{-1000}{2}} \cdot 2 \cdot \frac{10000}{2} \cdot \frac{-3000}{2} \cdot \frac{5000}{2} = 1 \]

\[ \log_{2} 2 = \frac{\ln 2}{\ln 8} = \frac{\ln 2}{3 \ln 2} = \frac{1}{3} \]

(OPT, \ log_{8} 2 \ is \ the \ solution \ to \ 8^{x} = 2. \ Since \ 8^{1/3} = 2, \)

\[ \log_{6} 2 + \log_{6} 3 = \log_{6} 2 \cdot 3 = 1 \]

\[ \log_{2} \left(\frac{1}{16}\right) = \log_{2} \frac{1}{2^{4}} = \log_{2} 2^{-4} = -4 \log_{2} 2 = -4 \]

\[ \ln e^{\pi} = \pi \ln e = \pi \]

\[ \pi^{\frac{1}{2}} \ln e^{2} = 2 \pi \]
9. \( f(x) = \frac{x+5}{3x-4} \)

\[
\text{Domain } f = \mathbb{R} \setminus \left\{ \frac{4}{3} \right\}
\]

Let \( y = \frac{x+5}{3x-4} \implies 3xy - 4y = x + 5 \)

\[
\implies 3xy - x = 4y + 5 \implies x(3y-1) = 4y + 5 \implies x = \frac{4y + 5}{3y-1}
\]

So \( f^{-1}(y) = \frac{4y + 5}{3y-1} \)

We observe that the inverse function \( f^{-1} \) is defined only if \( 3y - 1 \neq 0 \)

That means \( y \neq \frac{1}{3} \)

So \( \text{Domain } (f^{-1}) = \mathbb{R} \setminus \left\{ \frac{1}{3} \right\} \)

(6) From hypothesis we have

\[ P(0) = 10000 \text{ and } \quad P(3) = 300 \cdot 10000 \]

Since \( P(t) = a \cdot e^{kt} \) we must determine \( a \) and \( k \).

\[ P(0) = a \cdot e^0 = a \implies a = 10000 \]

\[ P(3) = a \cdot e^{3k} = 300 \cdot 10000 \implies 10 \cdot 10000 = e^{3k} \]

\[ \implies e^{3k} = 30 \quad \text{Taking the logarithm} \implies 3k \ln e = \ln 30 \]

\[ \implies k = \frac{\ln 30}{3} \]

The population when \( t = 4 \) is \( P(4) = a \cdot e^{4k} = 10000 \cdot e^{\frac{4}{3} \ln 30} \)

Using the formula \( e^{\ln x} = x \implies P(4) = 10000 \cdot (30)^{\frac{4}{3}} \cdot 10000 \cdot (30)^{\frac{3}{3}} = 10000 \cdot 30 \cdot \sqrt[3]{30} \)
Denote \( f(x) = x^5 - 3x + 1 \). We want to show that this function has a zero between 0 and 1.

\[
\begin{align*}
  f(0) &= 0 - 3 \cdot 0 + 1 = 1 \\
  f(1) &= 1 - 3 + 1 = -1 \\
\end{align*}
\]

Since \( f \) is continuous we can apply the Intermediate Value Theorem. Since 0 is between -1 and 1 there exist a number \( c \) such that \( f(c) = 0 \).