There are 8 problems in this exam. The pages are printed on both sides. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** Feel free to consult the Psychic Friends Network if you can do so telepathically.

Leave all answers in exact form (that is, do not approximate π, square roots, and so on.)

You have 90 minutes to complete this exam.
1. The graph of a function $f$ is shown below.

(a) **3 points** List all points $-6 \leq x \leq 6$ where $f(x)$ is not continuous. If there are none, write “none”.

(b) **3 points** What is \( \lim_{x \to -3^+} f(x) \)? If it does not exist, write DNE.

(c) **3 points** Is $f(x)$ continuous from the left at $x = -3$?

(d) **3 points** What is \( \lim_{x \to 4} (f(x/2) - f(x + 1)) \)?
2. Let $h(x) = \sqrt{\frac{x-1}{x}}$.

(a) [3 points] What is the domain of $h(x)$?

(b) [3 points] Find two functions $f$ and $g$ so that $h = f \circ g$.

(c) [4 points] Write a formula for $h^{-1}(x)$.

3. (a) [3 points] If $5e^{3x} = 10$, what is $x$?

(b) [3 points] Solve $\ln(x^2) = 6$ for $x$. If there are no solutions, write “none”.

(c) [3 points] What is the inverse of the function $f(x) = x^2$, with $x < 0$? If the function has no inverse, write “no inverse”.
4. A box without a top is to be made from a rectangular piece of cardboard which is
16 inches by 20 inches by cutting out four equal squares of side length \(x\) inches from
each corner, and then folding up the flaps to form the sides of the box (see figure).

\[ \begin{array}{c}
\includegraphics[width=0.5\textwidth]{box.png}
\end{array} \]

(a)  \textbf{6 points}  Express the volume of the box \(V\) as a function of \(x\).

(b)  \textbf{3 points}  What is the domain of the function \(V(x)\)?

5.  \textbf{9 points}  The graphs of several functions \(f(x)\) are shown below. On the same set of
axes, sketch the graph of the function \(g(x)\) as indicated.

\[ \begin{array}{c}
\includegraphics[width=0.3\textwidth]{functions.png}
\end{array} \]

\[ g(x) = f^{-1}(x) \quad g(x) = f(x) + x \quad g(x) = -f(x/2) \]
6. Compute each of the following limits. If the limit is undefined, please distinguish between $+\infty$, $-\infty$, and a limit which does not exist (DNE).

(a) $3$ points $\lim_{x \to 2} xe^{x-2}$

(b) $3$ points $\lim_{h \to 0} \frac{(4 + h)^2 - 16}{h}$

(c) $3$ points $\lim_{x \to +\infty} \frac{2x^2 - 19x + 7}{x^2 - 49}$

(d) $3$ points $\lim_{x \to +\infty} \sin \left( \frac{\pi}{x} \right)$

(e) $3$ points $\lim_{x \to 9} \frac{3 + \sqrt{x}}{3 - \sqrt{x}}$
7. Let \( q(x) = \begin{cases} \frac{x-1}{x+2} & x < 1 \\ x+2 & x \geq 1 \end{cases} \)

(a) \(3\) points Calculate \( \lim_{x \to 1^-} q(x) \). If the limit does not exist, write DNE.

(b) \(3\) points Calculate \( \lim_{x \to 1^+} q(x) \). If the limit does not exist, write DNE.

(c) \(3\) points For what \( x \) is \( q(x) \) continuous?

8. \(8\) points The equation \( 1 + \sin \left( \frac{\pi}{4} x^2 \right) - 3x = 0 \) has exactly one solution for \( 0 \leq x \leq 5 \). Between what two (closest) whole numbers does the solution lie? You must fully justify your answer to receive credit.