Compute the volume of a sphere of radius $r$ using an integral.

**SOLUTION.** The sphere of radius $r$ can be obtained rotating the half circle graph of the function

$$y = \sqrt{r - x^2}, \quad x \in [-r, r].$$

about the $x$-axis.

The volume $V$ is obtained as follows:

$$V = \int_{-r}^{r} \pi(r^2 - x^2) \, dx = 2 \int_{0}^{r} \pi(r^2 - x^2) \, dx = \frac{4}{3} \pi r^3.$$