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Problems 1 & 2: True or false: (Circle the correct answers.) Let $a$, $b$, $c$ and $d$ be positive integers.

\begin{enumerate}
\item True (T) or False (F): For every positive integer $n$, the congruence classes $\mathbb{Z}_n$ always contain nonzero zero divisors.
\item True (T) or False (F): Every nonempty set of positive integers contains a largest element.
\end{enumerate}

**SOLUTION:**
1. (1) is FALSE for primes $n$.
2. (2) is FALSE for $\{1, 2, 3, 4, \ldots\}$.

Problem 3: Let $n$ be an integer $\geq 2$. Define what it means for the nonzero congruence class $[a]_n \in \mathbb{Z}_n$ to be a zero divisor.

**SOLUTION:** There exists a $b \in \mathbb{Z}$ such that $b \not\equiv 0 \mod n$ and $ab \equiv 0 \mod n$.

Problem 4: Determine all $x \in \mathbb{Z}$ that solve the linear congruence

$6x \equiv 9 \mod 15$.

**SOLUTION:** Since $(6, 15) = 3|9$, an equivalent equation is

$2x \equiv 3 \mod 5$.

Since $[2]_5^{-1} = [3]_5$, the solution is given as

$x = [9]_5 = [4]_5$.

Problem 5: Let $p$ be an odd prime, prove that $\varphi(2p) = p - 1$.

**SOLUTION:** $\varphi(2p) = \varphi(2)\varphi(p) = 1(p - 1)$. 