Problems 1 & 2: True or false: (Circle the correct answers.) Let $a$, $b$, $c$ and $d$ be positive integers.

T  F  (1) If there exist integers $r$ and $s$ such that $ra + sb = d$, then $d = \gcd(a, b)$.
T  F  (2) $\text{lcm}(a, b) > \gcd(a, b)$.

SOLUTION. (1) is FALSE. Counterexample: $2 \cdot 1 + 2 \cdot 1 = 2$, but $(1, 1) = 1$.
(2) is FALSE: If $a = b$, then $\text{lcm}(a, b) = \gcd(a, b)$.

Problem 3: Express 24 and 102 as products of primes and use this information to calculate $\gcd(12, 102)$ and $\text{lcm}(12, 102)$.

SOLUTION. $24 = 2^3 \cdot 3$ and $102 = 2 \cdot 3 \cdot 17$. Therefore $\gcd(12, 102) = 2 \cdot 3 = 6$ and $\text{lcm}(12, 102) = 2^2 \cdot 3 \cdot 17 = 204$.

Problem 4: Use the Euclidean algorithm to calculate the $\gcd$ of $-24$ and $-102$.

SOLUTION.

\begin{align*}
-102 &= 5(-24) + 18, \\
-24 &= (-2)18 + 12, \\
18 &= 1 \cdot 12 + 6
\end{align*}

and

\begin{align*}
12 &= 2 \cdot 6.
\end{align*}

So we conclude that $\gcd(-24, -102) = 6$, as expected.

Problem 5: How many elements does $G_8 = \mathbb{Z}_8^*$ contain? List them.

SOLUTION. $G_8$ has 4 elements, they are $[1]_8, [3]_8, [5]_8$ and $[7]_8$. 