Economics 362 – Practice 3

1. Suppose that the risk free interest rate is \( r_f = 4\% \) and the expected return on the market portfolio is \( \bar{r}_M = 7\% \). The variance of the market portfolio is \( \sigma^2_M = .25 \)

(a) Using CAPM, find the expected rate of return of a security having a \( \beta = 1.5 \).

(b) Suppose that a security has an expected return of 5%. Use CAPM to determine the \( \beta \) of the security and its covariance with the market portfolio.

Answer.

(a) According to CAPM the expected return should be

\[
E(r_i) = r_f + \beta(\bar{r}_M - r_f).
\]

In our case \( r_f = 4\% \), \( \bar{r}_M = 7\% \) and \( \beta = 1.5 \). Therefore

\[
r_i = 4 + 1.5(7 - 4) = 8.5\%.
\]

(b) If \( r_i = 5 \) then

\[
5 = 4 + \beta(7 - 4)
\]

and solving for \( \beta \) we obtain \( \beta = \frac{1}{3} \). To find the covariance remember that \( \beta = \frac{Cov(r_i, r_M)}{\sigma^2_M} \), so that

\[
Cov(r_i, r_M) = \beta \sigma^2_M.
\]

Therefore \( Cov(r_i, r_M) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12} \).

2. Here are returns and standard deviations for four investments.

<table>
<thead>
<tr>
<th>Investment</th>
<th>Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury bills</td>
<td>0.06</td>
<td>0</td>
</tr>
<tr>
<td>Stock P</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>Stock Q</td>
<td>0.145</td>
<td>0.28</td>
</tr>
<tr>
<td>Stock R</td>
<td>0.21</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Calculate the standard deviations of the following portfolios
(a) 50% in treasury bills, 50% in stock P.
(b) 50% each in Q and R, assuming the shares have
   i. perfect positive correlation
   ii. perfect negative correlation
   iii. no correlation

Answer.

(a) The general formula for a portfolio of two securities is
\[ \text{Var} (\alpha r_a + (1-\alpha) r_b) = \alpha^2 \sigma_a^2 + (1-\alpha)^2 \sigma_b^2 + 2\alpha (1-\alpha) \rho_{ab} \sigma_a \sigma_b. \]
If \( \alpha = 0.5 \) and security \( a \) is a treasury bill, \( \sigma_a = 0 \) and \( \rho_{ab} = 0 \). If security \( b \) is stock \( P \) then \( \sigma_b = 0.14 \). We conclude
\[ \text{Var} (\alpha r_a + (1-\alpha) r_b) = (0.5)^2 (0.14)^2 \]
and the standard deviation is
\[ \sigma = \sqrt{(0.5)^2 (0.14)^2} = 0.07. \]

(b) If \( \alpha = 0.5 \), security \( a \) is stock \( Q \), and security \( b \) is stock \( R \) then \( \sigma_a = 0.28, \sigma_b = 0.26 \).
   i. If \( Q \) and \( R \) have perfect positive correlation then \( \rho_{ab} = 1 \) then
\[ \text{Var} (\alpha r_a + (1-\alpha) r_b) = (0.5)^2 (0.28)^2 + (0.5)^2 (0.26)^2 + 2 (0.5)^2 \times 1 \times 0.28 \times 0.26 = 0.0729 \]
and the standard deviation is
\[ \sigma = \sqrt{0.0729} = 0.27. \]
   ii. If \( Q \) and \( R \) have perfect negative correlation then \( \rho_{ab} = -1 \) then
\[ \text{Var} \left( \frac{1}{2} r_a + \frac{1}{2} r_b \right) = (0.5)^2 (0.28)^2 + (0.5)^2 (0.26)^2 - 2 (0.5)^2 \times 1 \times 0.28 \times 0.26 = 0.0001 \]
and the standard deviation is
\[ \sigma = \sqrt{0.0001} = 0.01. \]
iii. If Q and R have zero correlation then \( \rho_{ab} = 0 \) then

\[
Var \left( \frac{1}{2} r_a + \frac{1}{2} r_b \right) = (0.5)^2 (0.28)^2 + (0.5)^2 (0.26)^2 = 0.0365
\]

and the standard deviation is

\[
\sigma = \sqrt{0.0365} = 0.19105
\]

3. True or false?

(a) The CAPM implies that if you could find an investment with a negative beta, its expected return would be less than the risk-free interest rate.

(b) The expected return of an investment with a beta of 2 is twice as high as the expected return on the market.

(c) If a stock lies below the securities market line, it is undervalued.

Answer.

(a) True. According to CAPM the expected return \( E(r_i) \) of a security with beta \( \beta_i \) is

\[
E(r_i) = r_f + \beta_i (r_M - r_f)
\]

Therefore, if \( r_M - r_f > 0 \) (as it is normally the case), the expected return of a security with a negative \( \beta_i \) should be less than \( r_f \).

(b) False. In this case the expected return is

\[
E(r_i) = r_f + 2 \left( r_M - r_f \right) = 2r_M - r_f
\]

which is less than \( 2r_M \) whenever \( r_f > 0 \).

(c) False. The fair price should be \( \hat{P}_t = \frac{E(P_{t+1})}{1 + r_f + \beta_i (r_M - r_f)} \). If a stock lies below the security market line then \( E(r_i) < r_f + \beta_i (r_M - r_f) \), so the actual price is

\[
P_t = \frac{E(P_{t+1})}{1 + E(r_i)} > \hat{P}_t.
\]

Therefore the security is overpriced.

4. True or false?
(a) Investors demand higher expected rates of return on stocks with more variable rates of return.

(b) The CAPM predicts that a security with $\beta = 0$ will offer a zero expected return.

(c) An investor who puts $10,000 in Treasury bills and $20,000 in the market portfolio will have a beta of 2.

(d) Investors demand higher expected rates of returns from stocks with returns that are very sensitive to fluctuations in the stock market.

Answer.

(a) False. Investors demand higher expected rates of return on stocks with returns more correlated to the return on the market portfolio (or other macroeconomic risk factors).

(b) False. A security with a beta of zero will offer the risk-free rate of return.

(c) False the beta will be

$$\beta = \frac{1}{3} \times 0 + \frac{2}{3} \times 1 = \frac{2}{3}.$$ 

(d) True.