Solutions to Practice Final Exam

1. Using the Black Scholes formula, compute the price of a call option with strike price $X = 45$ expiring in 156 days. The current stock price is $S = 44.375$, the riskless rate is $r = 0.07$ and the volatility is $\sigma = 0.31$.

In this case $T = \frac{156}{365}$. Applying the formula:

$$d_1 = \frac{\log \left( \frac{S}{X} \right) + rT}{\sigma \sqrt{T}} + \frac{1}{2} \sigma \sqrt{T} \quad d_2 = d_1 - \sigma \sqrt{T}$$

$$d_1 = \frac{\ln \left( \frac{44.375}{45} \right) + 0.07 \times \frac{156}{365}}{0.31 \times \sqrt{\frac{156}{365}}} + \frac{1}{2} \times 0.31 \times \sqrt{\frac{156}{365}} = 0.17994$$

$$d_2 = \frac{\ln \left( \frac{44.375}{45} \right) + 0.07 \times \frac{156}{365}}{0.31 \times \sqrt{\frac{156}{365}}} - \frac{1}{2} \times 0.31 \times \sqrt{\frac{156}{365}} = -0.022722$$

we obtain

$$N (d_1) = \int_{-\infty}^{0.17994} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx = 0.5714$$

$$N (d_2) = \int_{-\infty}^{-0.022722} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx = 0.49094$$

and the price of the option is

$$c = 44.375 \times 0.5714 - 45e^{-0.07\times\frac{156}{365}} \times 0.49094 = 3.9147.$$ 

2. Consider the following combination:

Buy a put option with strike price $X = 100$.
Buy a call option with strike price $X = 100$.
Sell a call option with strike price $X = 200$.

The final payoff is given by the following table which is exactly the desired payoff.
3. We have $r = 0.05$, earnings per period equal to 400000, dividends equal earnings.

(a) If there are 100000 shares then each share receives a dividend of 4 is each period. The price $p_0$ of each share is

$$p_0 = \sum_{t=1}^{\infty} \frac{4}{(1.05)^t} = \frac{4}{0.05} = 80.$$  

(b) The company now announces that it will use the earnings of period 1 to buy back shares.

i. Let $N$ be the number of shares which are repurchased. The price $p_1$ has to satisfy

$$p_1 = \frac{400000}{(100000 - N) 0.05}$$

Furthermore, the amount spent has to be 400000. Therefore $p_1$ and $N$ must satisfy

$$Np = 400000$$

The solution of this system is $N = 4761.9, p_1 = 84$. Therefore, from period 1 on there will be only \(100000 - 4761.9 = 95238.1\) shares.

ii. To compute $p_0$ after the company decides to change the policy, observe that now the firm does not pay dividends next period. Therefore the current price is given by

$$p_0 = \frac{p_1}{1 + r}$$
which in our case works out to

\[ p_0 = \frac{84}{1.05} = 80. \]

Notice that this conclusion was to be expected, given the Modigliani-Miller theorem on the irrelevance of dividends.