The Wald Tests for Testing Hypotheses

Eco321: Econometrics

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An Example: The Determinants of Wage

The Econometric Model

\[ \log(\text{Wage}) = \beta_0 + \beta_1 \text{Age} + \beta_2 \text{Education} + \beta_3 \text{Female} + \epsilon \]
Matlab Commands

The dependent and explanatory variables

% open dataset
data = csvread('wages2.csv') ;

% Working People
data = data (data(:,10)>0, :);

% Dependent Variable: Wage
y = data(:,10) ; LogWage = log(y) ;

% Explanatory Variables
Age = data(:,3) ; Education = data(:,5) ; Female = data(:,6) ;
X = [Age Education Female] ;
Matlab Commands

Estimation by OLS

% Regress LogWage on X
Out = regstats (LogWage, X, 'linear') ;

% betahat and variance-covariance matrix
betahat = Out.beta ; Var = Out.covb ;
The Wald Test

Testing Hypothesis

\[ H_0 : R \beta = r \]

where \( R \) is a \( q \times k \) matrix

The Test Statistic

\[ W = (R \hat{\beta} - r)' [R \text{Var}(\hat{\beta}) R]'^{-1} (R \hat{\beta} - r) \overset{\sim}{\sim} \chi_q^2 \]
Example 1

\[ H_0 : \beta_2 = 0 \]

\[
\begin{bmatrix}
0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\beta_0 \\
\beta_1 \\
\beta_2 \\
\beta_3 \\
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
r \\
\end{bmatrix}
\]
The Wald Test

Matlab Commands

% R and r matrices
R = [0 0 1 0] ; r = 0 ;

% The Wald test statistic
diff = R*betahat - r ; Vdiff = R*Var*R' ;
W = diff' * inv(Vdiff) * diff ;

% The rejection region
df = size(R,1); alpha = 0.05 ;
Reject = chi2inv(1-alpha, df);
Example 2

\[ H_0 : \beta_2 = \beta_3 \]

\[
\begin{bmatrix}
0 & 1 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
\beta_0 \\
\beta_1 \\
\beta_2 \\
\beta_3
\end{bmatrix}
= \begin{bmatrix}
0
\end{bmatrix}
\]
Example 3

\[ H_0 : \beta_1 = 0 \text{ and } \beta_2 = 0 \]

\[
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\beta_0 \\
\beta_1 \\
\beta_2 \\
\beta_3
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]
The Wald Test

Test for structural change

For Men:

$$\log(Wage_M) = X_M \beta_M + \epsilon_M$$

For Women:

$$\log(Wage_W) = X_W \beta_W + \epsilon_W$$

Testing Hypothesis

$$H_0 : \beta_M = \beta_W$$

where $\beta_M$ and $\beta_W$ are $k \times 1$ matrices
The Wald Test

- Divide the whole sample into men’s and women’s samples.
- Run Regressions, separately.
- Find $\hat{\beta}_M$, $\text{Var}(\hat{\beta}_M)$ and $\hat{\beta}_W$, $\text{Var}(\hat{\beta}_W)$

The Test Statistic

$$W = (\hat{\beta}_M - \hat{\beta}_W)'[\text{Var}(\hat{\beta}_M) + \text{Var}(\hat{\beta}_W)]^{-1}(\hat{\beta}_M - \hat{\beta}_W) \sim \chi^2_k$$

See the Professor’s class handout, ”Comparing Women’s and Men’s Wage Regressions”, on Blackboard.