

MAT 127 Final I, Practice Exam

Last Name: _____ First Name: _____ Student ID: _____

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|----------------|-----------|-----------|-----------|-----------|-----------|-----------|--------------|
| <i>Problem</i> | 1 | 2 | 3 | 4 | 5 | 6 | Total |
| <i>Points</i> | 10 | 10 | 40 | 10 | 10 | 20 | 100 |
| <i>Scores</i> | | | | | | | |

This midterm has five problems, weighted as shown. Please show your work – full credit may not be given if only the answers appear. **No calculators or books will be allowed on this test.** When calculating indefinite integrals, the answers should be in explicit forms, unless otherwise stated.

1. Sketch the directional field of the differential equation

$$\frac{dy}{dx} = xy - x^2.$$

Sketch the solution curves that satisfies the initial conditions: i) $y(0) = 0$; ii) $y(0) = 1$.

2. Use Euler's Method with step size 0.1 to estimate $y(0.4)$, where y is the solution of the following initial-value problem

$$\frac{dy}{dx} = x + y^2, \quad y(0) = 0.$$

3. Solve the following **separable** differential equations:

(a) $\frac{dx}{dt} = 1 - t + x - tx.$

$$(b) \frac{dy}{dt} = \frac{e^y \sin^2 t}{y \sec t}$$

(c) $\frac{dr}{dt} + 2tr = r, \quad r(0) = 5$

$$(d) \quad x \ln x = y(1 + \sqrt{3 + y^2})y', \quad y(1) = 1.$$

4. Find the orthogonal trajectories of the family of curves: $y = e^{kx}$.

5. The half-life of cesium-137 is 30 years. Suppose we have a 100-mg sample. Find the mass that remains after t years.

6. Solve the following **second order** differential equations.

(a) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 4y = 0.$

$$(b) \quad y'' + 16y = 0, \quad y(\pi/4) = -3, \quad y'(\pi/4) = 4.$$