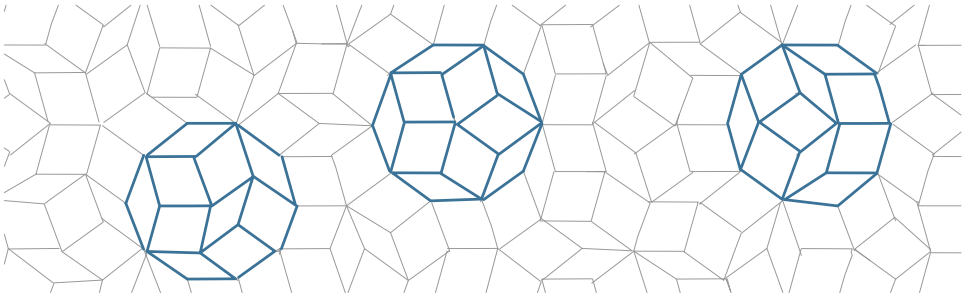
**The Penrose Tiling**

The Penrose tiling on this pavement is part of a scheme for filling up the entire plane with copies of the "fat rhombus" (angles 108° and 72°), and the "thin rhombus" (angles 144° and 36°). Its two most important properties are

- The tiling is not periodic. There is no way to slide the picture, in any direction, so that it matches itself perfectly.
- Nevertheless, any finite patch of the tiling, no matter how large, will appear in exact copies of itself over and over again as we look farther and farther out in the plane.

This tiling is one of a set of aperiodic tilings investigated by the British mathematician Roger Penrose (now Sir Roger) in the 1970s.

The most elegant way to describe the Penrose tiling is as the projection onto the plane of a certain specific surface in 5-dimensional space. This surface is made up of squares parallel to the coordinate planes, with adjoining squares perpendicular to each other (all this in 5-space). There are 10 possible orientations for such a square; projected these give the two rhombic shapes, with 5 possible orientations for each of them.



In the surface in 5-space, whenever a vertex meets exactly three squares, these must be relatively positioned like three faces of a cube. When the three are projected into the plane, they look like the isometric perspective image of an ordinary 3-dimensional cube. But this interpretation cannot be extended to neighboring tiles. Even though those may be part of another cube, the cubes, interpreted in 3-space, form an "impossible" configuration. Some instances of this phenomenon, which forces our perception of the the tiling to continually shift from one interpretation to another, are highlighted in blue in the figure above.

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