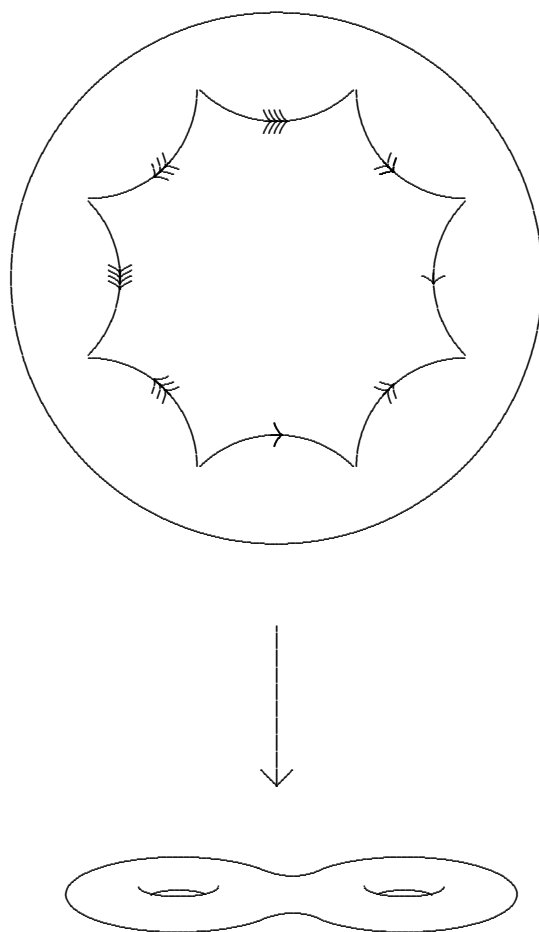


Ideas for the Simons Wall Project

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March 5, 2010

Uniformization of a surface of genus 2:



Index theorem for coupled Dirac operator:

$$\not{D} : \Gamma(\mathbb{S}_+ \otimes E) \rightarrow \Gamma(\mathbb{S}_- \otimes E)$$

$$\dim \ker(\not{D}) - \dim \ker(\not{D}^*) = \int_{M^{4k}} \hat{A}(M) \smile ch(E)$$

Generalized Gauss-Bonnet theorem:

$$\chi(M^{2n}) = \frac{1}{(8\pi)^n n!} \int_M \underbrace{R_{ab}^{ij} \cdots R_{cd}^{kl}}_n \varepsilon^{ab \cdots cd} \varepsilon_{ij \cdots kl} d\mu$$

Classical Gauss-Bonnet theorem:

$$\chi(M^2) = \frac{1}{2\pi} \int_M K dA$$

$$\chi(M) = -4$$


Einstein's gravitational field equations:

$$R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Hodge theorem:

$$H^p(M) = \{\varphi \in \Omega^p(M) \mid d\varphi = 0, \ d \star \varphi = 0\}$$

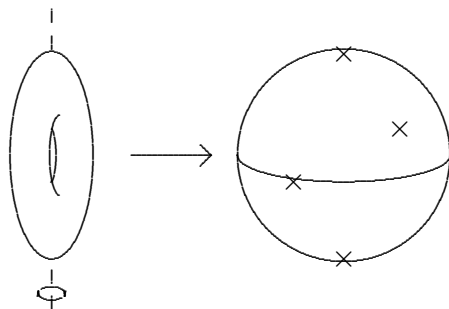
Dirac Equation:

$$\begin{aligned}(-i\hbar\gamma^\mu\nabla_\mu + mc)\psi &= 0 \\ \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu &= g^{\mu\nu}\end{aligned}$$

Seiberg-Witten Equations:

$$\begin{aligned}\not{D}_A \psi &= 0 \\ F_A^+ &= -\frac{1}{2} \psi \odot \bar{\psi}\end{aligned}$$

Elliptic curve, and uniformizing elliptic integral:



$$F(z) = \int_{z_0}^z \frac{d\zeta}{\sqrt{\zeta(\zeta - 1)(\zeta - \lambda)}}$$