Ideas for the Simons Wall Project

Claude LeBrun
Department of Mathematics
Stony Brook

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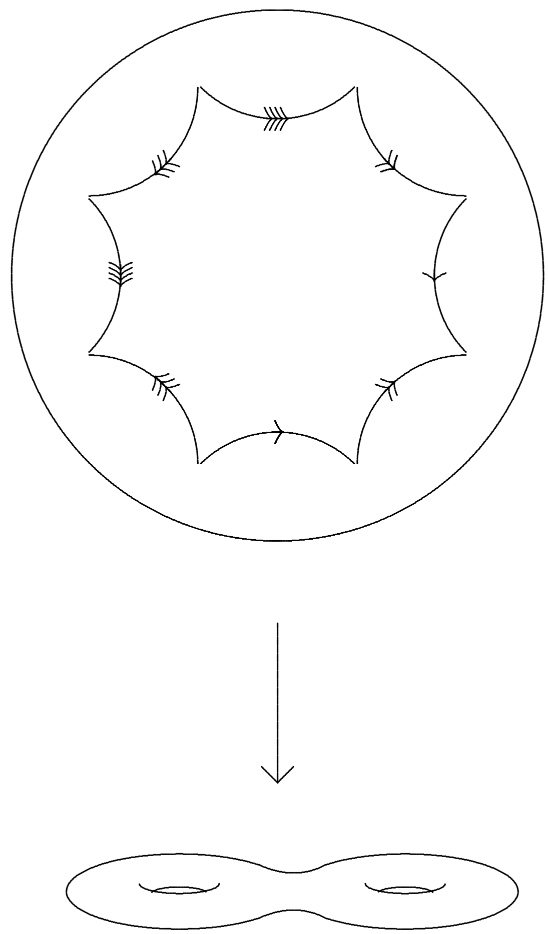

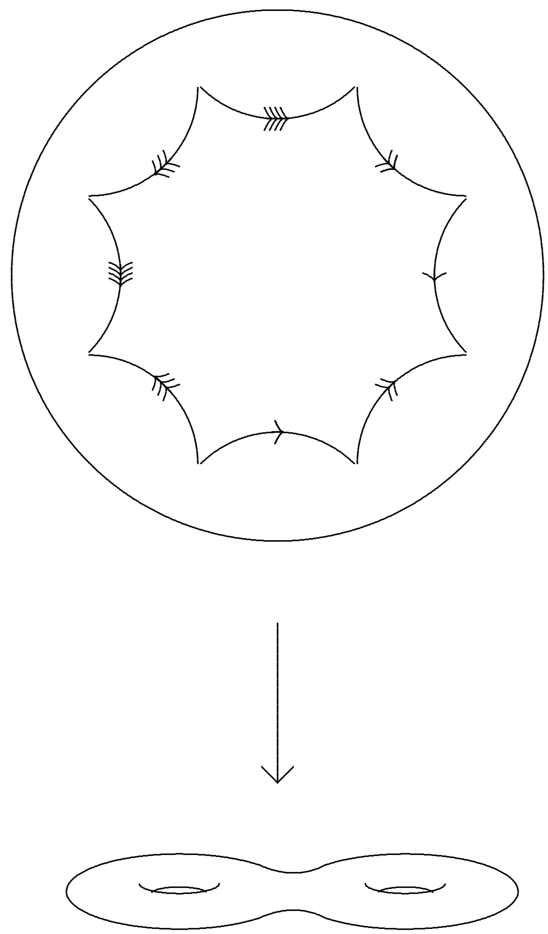

Explanation:

This diagram depicts the uniformization of a surface of genus 2.
It shows that such such a surface can be given a metric of constant
Gauss curvature by displaying it as a quotient of hyperbolic 2-space
by a group of isometries.

$$\partial\!\!\!/ : \Gamma(\mathbb{S}_+ \otimes E) \to \Gamma(\mathbb{S}_- \otimes E)$$

$$\dim\ker(\partial\!\!\!/) - \dim\ker(\partial\!\!\!/^*) = \int_{M^{4k}} \hat{A}(M) \smile ch(E)$$

$$\not{\partial} : \Gamma(\mathbb{S}_+ \otimes E) \to \Gamma(\mathbb{S}_- \otimes E)$$

$$\dim\ker(\not{\partial}) - \dim\ker(\not{\partial}^*) = \int_{M^{4k}} \hat{A}(M) \smile ch(E)$$

$$\hat{A} = 1 - \frac{p_1}{24} + \frac{7p_1^2 - 4p_2}{5760} + \cdots$$

$$ch = \text{rank} + \frac{c_1}{2} + \frac{c_1^2 - 2c_2}{2} + \cdots$$

Explanation:

This is the Atiyah-Singer index formula for the twisted Dirac operator, where E is a complex vector bundle over a smooth compact spin manifold of dimension $4k$. The additional formulæ on this page give more information on the terms involved. This arguably makes the display too busy, however.

$$\chi(M^{2n}) = \frac{1}{(8\pi)^n n!} \int_M \underbrace{R_{ab}^{ij} \cdots R_{cd}^{k\ell}}_{n} \varepsilon^{ab\cdots cd} \varepsilon_{ij\cdots k\ell} \, d\mu$$

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Explanation:

This is the generalized Gauss-Bonnnet theorem. It expresses the Euler characteristic of any smooth compact even-dimensional manifold M as a curvature integral that can be computed using any Riemannian metric g on M.