MAT 160 Spring 2010

Homework. Due April 20.

1. A mathematician organizes a lottery in which the prize is an infinite amount of money. When the winning ticket is drawn, and the jubilant winner comes to claim his prize, the mathematician explains the mode of payment: "1 dollar now, 1/2 dollar next week, 1/3 dollar the week after that..." (from University of Utah Math Department joke pages). Explain the joke.

2. Here's another lottery problem. If money can earn 5% interest, the *present value* of \$100 payable a year from now is $\frac{100}{1.05}$ dollars, since that sum would be worth \$100 by the end of the year. Suppose the mathematician above tells the winner: "You'll get \$100 today, \$100 a year from now, and \$100 every year forever." What is the present value of that award?

3. Evaluate $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \cdots$.

4. Suppose the "Solar system" of wine ageing is modified so that at the end of the year $\frac{1}{3}$ of the wine is sold and replaced with new wine. What is the average age of the wine that was sold?

5. Modify the maneuver we used to settle the "Solar" problem to prove:
$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots = \ln(2).$$
(Hint: you will integrate instead of differentiating).

6. We proved using Fourier series that
$$1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots = \frac{\pi^2}{8}.$$
Use this to prove Euler's equation:
$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots = \frac{\pi^2}{6}.$$