MAT 160 Spring 2010

Homework. Due March 23.

Euler's "7 Bridges of Königsberg" problem involves parity, but it is also a problem about topology: it involves paths and crossings without any reference to lengths or angles. Only the *relative position* of the elements matters. Euler described this field as *Analysis situs*, the analysis of location; since around 1900 we know it as topology.

1. You want to connect 3 utilities (say: Gas, Electricity, Water) to 3 houses (say: Jones, Smith, Taylor). Can this be done without any of the lines crossing?

2. You want to join each of 5 points to each of the other 4. Can this be done without any of the lines crossing?

3. Venn diagrams: We can represent propositions P, Q, R by connected regions (disks, for instance) A, B, C in the plane so that the intersections of these regions and their complements A', B', C'graphically realize all the possible logical expressions that can be written with P, Q and R and their negations $\sim P, \sim Q, \sim R$. For example $P \wedge (\sim Q \vee R)$ corresponds to $A \cap (B' \cup C)$. Check that the intersections of 3 disks in general position with each other and with their complements determine 8 distinct connected regions. These correspond to the 8 three-character monomials $(P \vee Q \vee R), (P \vee Q \vee \sim R), ..., (\sim P \vee \sim Q \vee \sim R)$.

   Show that this cannot be done for 4 propositions P, Q, R, S.

   Suppose you can work with 3-dimensional balls rather than 2-dimensional disks. Can you handle 4 propositions?

   How about 5 propositions?

4. Show that any continuous function $f\colon [0, 1] \to [0, 1]$ from the closed interval to itself must have a fixed point: some x, $0 \le x \le 1$ such that $f(x) = x$.