

MAT 160 Spring 2010

Homework. Due March 9.

The Pigeonhole Principle: If $n + 1$ objects are distributed among n boxes, one of the boxes will contain at least 2 objects. More generally, if $kn + 1$ objects are distributed among n boxes, one of the boxes will contain at least $k + 1$ objects.

1. Prove that for any set of five points in the interior of a square of side length one, there is at least one pair of points whose distance is less than $\sqrt{2}/2$.
2. In a chessboard, remove two opposite corners. Is it possible to cover this board with domino tiles whose size is exactly two board squares?
3. Given any ten integers, show that there is a pair of these integers whose difference is divisible by 9.
4. Fifty-one trees are planted inside a square field with sides of length 100 feet. Show that some set of 3 of these trees must be contained in a square with sides of length 20 feet.
5. A certain dorm has 25 people in it. There are four kitchens in the dorm, and each person is randomly assigned to a kitchen. Show that there is a group of 7 people who are assigned to the same kitchen.