

MAT511 homework, due Nov 18, 2009

- (1) Give a proof using the Pigeonhole Principle: If five points are in or on a square of side length 1, then at least two points are no farther apart than $\frac{\sqrt{2}}{2}$. Start by drawing a picture.
- (2) Define B^A to be the set of all functions from A to B . Show that if A and B are finite, then B^A is finite. What is the cardinality of B^A in terms of the cardinalities of A and B ?
- (3) Prove that the two intervals $(0, 1)$ and $(-47, 47)$ have the same cardinality.
- (4) Prove that the interval $(0, 1)$ has the same cardinality as the set \mathbf{R} of real numbers.
- (5) Prove that if A is an infinite set and $A \subseteq B$, then B is an infinite set.
- (6) Let $3\mathbf{N}$ be the set of natural numbers that are multiples of 3, that is $3\mathbf{N} = \{3, 6, 9, 12, \dots\}$. Prove that $\mathbf{N} - 3\mathbf{N}$ is a countably infinite set.
- (7) Show that there is always a one-one correspondence between a set A and a *subset* of its power set $\mathcal{P}(A)$.
- (8) Reproduce the proof given in class that there can never be a one-one correspondence between A and $\mathcal{P}(A)$.