MAT511 homework, due Nov 11, 2009

(1) For each of these functions $f(x)$ find the maximum domain of definition $\mathcal{D} \subset \mathbf{R}$. Then restrict the domain to $\mathcal{D}'$ on which f is one-one (choose $\mathcal{D}'$ as large as possible. What is the range $\mathcal{R}$ of the restricted function? Give a formula for the inverse function $g(y)$ mapping $\mathcal{R}$ to $\mathcal{D}'$.

(a) $f(x) = \sqrt{x^2 - 4}$
(b) $f(x) = \frac{1}{x^2+3}$
(c) $f(x) = \sqrt{x^2 - x}$

(2) List the first five terms of the sequence $(n = 1, 2, 3, \dots)$

(a) $x_n = \frac{1}{n!}$
(b) $x_n = \frac{cos(n)}{n}$
(c) $x_n = 1 - \frac{1}{2^n}$

(3) Prove carefully that sequences (1) and (2) converge to zero (using the definition) and that (3) converges to 1.

(4) Prove carefully that if x_n and y_n are sequences $(n = 1, 2, 3, \dots)$ which converge to limits L and M, then the sequence $x_n + y_n$ converges to $L + M$. You must use the definition of "converges to limit"!

(5) Give an example of sequences x_n and y_n $(n = 1, 2, 3, \dots)$ which do not converge, but such that the sum $x_n + y_n$ converges.