

Work all four proofs *using induction*.

- (1) Use induction to prove that, for all natural numbers n ,

$$1 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

- (2) Use induction to prove that, for all natural numbers n ,

$$1 + 4 + 7 + \dots + (3n - 2) = \frac{n(3n - 1)}{2}$$

- (3) Use induction to prove that for any natural number n , $n^3 + 5n + 6$ is divisible by 3.

- (4) If a set A has n elements, use induction to prove that its power set $\mathcal{P}(A)$ has 2^n elements.

- (5) Let P_1, P_2, \dots, P_n be n points in a plane, no three of which are collinear. Prove (by induction) that the number of line segments joining all pairs of points is $(n^2 - n)/2$.

