MAT511 homework, due Dec 9, 2009

(1) Give a careful proof of the proposition:

If A is an infinite set, and X is any other set disjoint from A, then $A \cup X$ is infinite.

[You must exhibit a 1-1 correspondence between $A \cup X$ and a proper subset of $A \cup X$.]

(2) Give a careful proof of the proposition:

If A is denumerable and $B \sim A$, then B is denumerable.

[You must exhibit an explicit 1-1 correspondence $\mathbf{N} \to B$.]

(3) Give a careful proof of the proposition:

If A is a finite set, then there does not exist a 1-1 correspondence f between A and a proper subset of A.

[You must use the definition of *finite* (p.224) and show that the existence of f would imply the existence of a 1-1 correspondence $\mathbf{N}_m \to \mathbf{N}_n$ for some $m \neq n$, and show why this is impossible.]

(4) Give a careful proof of the proposition:

The set $\mathbf{R}$ of real numbers has the same cardinality as the set $\mathbf{R} - 0$ of nonzero real numbers.

[We did this in class. Write each of them as a union of half-open intervals, and set up an *explicit* 1-1 correspondence between pairs of intervals, one in R, one in $\mathbf{R} - 0$.]