MAT 342 Fall 2010
Review for Midterm 2

Section 21. Understand how the Cauchy-Riemann equations at z follow from differentiability at z, and that $f'(z) = u_x + iv_x$, where $f(x + iy) = u(x, y) + iv(x, y)$. Exercises 1, 2 p.71.

Sections 24-25. Understand that "analytic at z" means "has a complex derivative at each point of some neighborhood of z." Remember that an open set S is *connected* if any two points can be joined by a finite chain of line segments lying in S, and that a *domain* is an open set that is connected. Then understand how to prove, and be able to use, the Theorem (p.74): *If $f'(z) = 0$ everywhere in a domain D, then f must be constant throughout D.* Exercises 1, 2, 4 p.77.

Section 26. Understand why if $f(x + iy) = u(x, y) + iv(x, y)$ is analytic, then u and v are *harmonic*: they satisfy Laplace's equation, i.e. $u_{xx} + u_{yy} = 0$, $v_{xx} + v_{yy} = 0$. Be able to calculate the *harmonic conjugate* of a harmonic function (Example 5 p.81). Exerises 1, 3, 4 p.82.

Section 29. Understand that the exponential map $e^{x+iy} = e^x(\cos y + i \sin y)$ takes each horizontal strip $y_0 < y \leq y_0 + 2\pi$ one-one onto the complement of 0. Exercises 1, 2, 11 p.92.

Sections 30, 31. Understand that since the exponential map covers the plane infinitely often, every non-zero z has infinitely many possible logarithms, and that they all differ by multiples of $2\pi i$. Know how to compute Log (z) (the *principal value* of the logarithm function) and understand why it is discontinuous at every point of the negative x-axis. Understand why you can define a *branch* of log by choosing an angle α and stipulating $\log(re^{i\theta}) = \ln r + i\theta, \alpha < \theta \leq \alpha + 2\pi$, and that any brach of log is analytic except where it is discontinuous. Exercises 1-6 p.97 and 1, 2 p.100

Section 33. Understand that complex *exponentiation* is defined by $z^c = e^{c \log z}$, where log is the multiple-valued logarithm, so that z^c is *in general* multiple-valued. And that writing $z^c = e^{c \text{ Log z}}$ (Log the principal value) gives the principal value of z^c, and that this function may be discontinuous. Exercises 1, 2, 3 p.104.

Section 34. Know the Euler formula: $e^{iz} = \cos z + i \sin z$ for *complex* z, and use it to define sin and cos in terms of the exponential function. Notice that sin and cos are entire, and that $\tan, \cot, \sec, \csc$ can be defined as usual and that the derivatives of these functions satisfy the usual differentiation formulas. Exercises 1, 2, 3 p.108.

Section 37, 38. Understand that the integral of a complex-valued function of a real variable is obtained by integrating the real and imaginary parts separately. Exercise 2 p.121.

Section 39, 40. A *contour* C is a piecewise smooth arc. Suppose there are n smooth pieces $C_1, \ldots, C_n$, parametrized by $\gamma_i : [a_i, b_i] \to \mathbf{C}$ (here $\mathbf{C}$ = the complex numbers). Then defining

$\int_C f(z)\ dz$ by

$$\int_C f(z)\ dz = \sum_{i=1}^{n} \int_{a_i}^{b_i} f(\gamma_i(t))\ \gamma_i'(t)\ dt$$

gives a number independent of parametrizations. Note the factors $\gamma_i'(t)$ which are crucial. Be able to evaluate contour integrals by choosing parametrizations and carrying out the real (dt) integrations. Exercise 2 p.125. Understand Exercise 5 p.125 (chain rule)! Try to work all the exercises on pages 135-136.