My NAME is:

Problem	1	2	3	Total
Score				

MAT 341
Applied Real Analysis
Midterm 2

November 14, 2011   Total score = 105.

This test is open book: Powers "Boundary Value Problems" may be consulted. No other references or notes may be used. Students may use graphing calculators like TI-83, 84, 85, 86; but they may NOT use calculators with Computer Algebra Systems, like TI-89. SHOW ALL YOUR WORK! When using Powers or your calculator be sure to report it, e.g. "from calculator," "from Powers page x."

1. (35 points) Solve the Heat Equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$$

on the semi-infinite rod $0 \leq x$ subject to the boundary condition

$$\frac{\partial u}{\partial x}(0,t) = 0 \qquad 0 < t$$

(rod is insulated at $x = 0$) and with initial temperature distribution

$$u(x,0) = f(x) = \begin{cases} \frac{1}{2} & 0 < x < 2 \\ 0 & x > 2 \end{cases} \quad .$$

2. (35 points) The wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \qquad 0 < x < 4 \qquad 0 < t$$

with boundary conditions

$$u(0,t) = u(4,t) = 0 \qquad 0 < t$$

governs the vibrations of a wire of length 4. At $t = 0$ the wire is stretched to the shape

$$u(x,0) = f(x) = \begin{cases} x & 0 < x < 1 \\ \frac{4}{3} - \frac{x}{3} & 1 < x < 4 \end{cases},$$

and released, with no initial velocity ($\frac{\partial u}{\partial t}(x,0) = 0$). Use d'Alembert's method to determine the exact shape of the wire at time $t = \frac{1}{c}$.

3. (35 points) Solve the eigenvalue/eigenfunction problem

$$\phi''(x) + \lambda^2\phi(x) = 0, \qquad 0 < x < 1$$

$$\phi'(0) - 3\phi(0) = 0, \qquad \phi(1) = 0$$

(this could occur in the study of the heat equation in a bar of length 1 with convection at the 0-end, and temperature at the 1-end held at 0). I.e., determine the eigenvalues $\lambda_1, \lambda_2, \ldots$ and the corresponding eigenfunctions $\phi_1(x), \phi_2(x), \ldots$.

End of Examination.