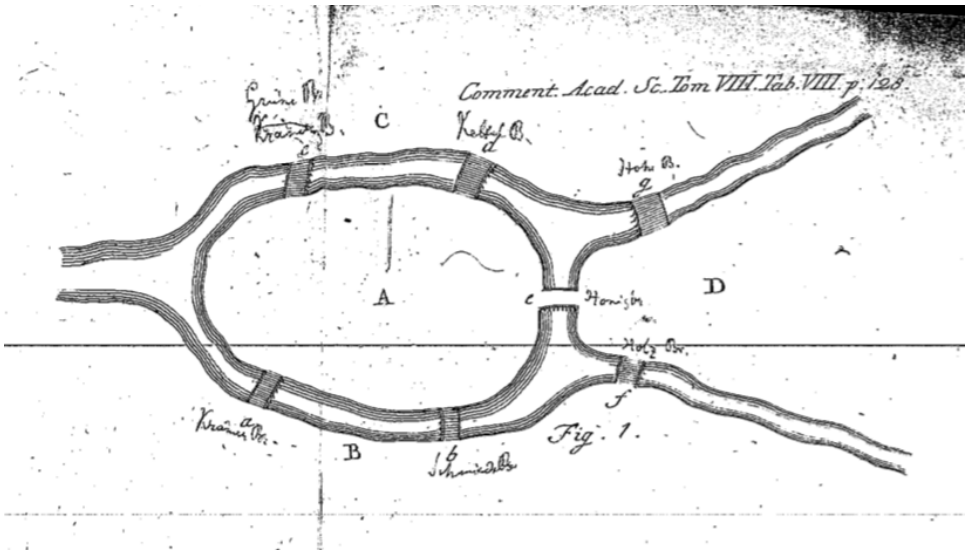
Euler: The solution of a problem pertaining to the Geometry of position.

§. I.

Besides that part of Geometry that concerns quantities, and which has been thoroughly studied for all time, another part before then unknown was first mentioned by *Leibniz*, who called it the Geometry of position. ...



§. 2. The problem, which I believe is well known, was the following: in Königsberg in Prussia is an island A called der Kneiphof, and the river surrounding it divides into two branches, as can be seen from the figure: the branches of this river are crossed by seven bridges, a, b, c, d, e, f, and g. Concerning these bridges the question was asked, whether someone would be able to devise such a path, so as to cross each bridge once and not more than once. ...

§. 4. My method starts with describing in a convenient way the path across the bridges; for this I use capital letters A, B, C, D for each of the regions separated by the river. Thus if someone crosses from region A to region B by bridge a or b, I will denote this crossing by the letters AB, of which the former represents the region the traveler is leaving, and the latter gives the region where the crossed bridge arrives. If originally the traveler were to go from region B to region D across bridge f, this crossing would be represented by the letters BD; two successive crossings AB and BD are

represented by the three letters ABD, since the central B designates both the region where the first crossing arrives and the region which the other crossing leaves.

§. 5. Similarly if the traveler goes from region D to region C over bridge g, I will denote these three successive crossings by the four letters ABDC. By these four letters ABDC it is understood that the traveler starting first in region A has crossed into region B, from there has moved into region D, and from from there has arrived in C: since in fact these regions are separated one from the other by the river, it will be necessary for the traveler to have crossed three bridges. This path over four bridges one after the other will be denoted by five letters; and if the traveler crosses any number of bridges, his trip will be denoted by a number of letters which is one more than the number of bridges. Hence to describe the path over the seven bridges will require eight letters.

§. 6. In this description I do not specify which bridges are crossed by the path, but if the same crossing can be made from one region to another over more than one bridge, it does not matter which one is crossed, as long as it leads to the correct region. From which we can understand that if a path can be drawn over the seven bridges, so that each one is crossed once and none is crossed twice, this path can be represented by eight letters in such a way that the immediate juxtaposition of the letters A and B occurs twice, since there are two bridges a and b connecting the regions A and B; similarly the juxtaposition of letters A and C must also occur twice in that series of eight letters; furthermore the juxtaposition of the letters A and D must occur once; similarly it is necessary for the juxtaposition of letters B and D, as well as that of C and D, to occur once.

§. 7. The question thus reduces to whether with the four letters A, B, C, D one can form a series of eight letters, in which all those juxtapositions occur exactly as often as was described. Before applying efforts to such an arrangement, we should determine we should search for such ways of disposing the letters or not. If in fact it could be shown that such an arrangement could not in any way be made, then any effort to carry it out would be useless. Therefore I sought a rule with the help of which both in this question and other similar ones it could be easily resolved, whether such an arrangement of letters could exist.

§. 8. To find this kind of rule I consider a single region A to which a

certain number a, b, c, d, etc. of bridges lead. Of these bridges I examine first just a, which leads to the region A; now if a traveler crosses this bridge either he has to have been in the region A before the crossing or he arrived in A after the crossing; whence in the manner of describing the path established above it is necessary for the letter A to occur once. If you imagine that three bridges a, b, c lead to the region A, and that the traveler crosses all three, then in the description of his movement the letter A is necessary twice, whether he started his path from A or not.

§. 9. Therefore in the case of crossing the bridges of Königsberg, since five bridges a, b, c, d, e lead to the island A, it is necessary that in the description of the path through those bridges the letter A occur three times. Furthermore the letter B, since three bridges lead to the region B, must occur twice; similarly the letter D must occur twice, and also the letter C twice. In the series of eight letters, which would describe the path across the seven bridges, the letter A should appear three times, and the letters B, C, and D each twice; which in a series of eight letters in all cannot be done. From which it is clear that such a path across the seven bridges of Königsberg cannot be set up.