Practice Midterm I

Problem 1. Show by induction that for any natural number $n \geqslant 1$ one has $3^n > n$.

Problem 2. Determine the set A of all x in $\mathbb{R}$ such that $|5.x + 2| < 8$.

Problem 3. Is the set $B = \left\{ \frac{1}{n^2 + 1}, n \in \mathbb{N} \right\}$ bounded above? bounded below? Does it have a least upper bound, a greatest lower bound?

Problem 4. Let $J_n = (1 - \frac{1}{n}, 2 + \frac{1}{n})$. Prove that $\bigcap_{n=1}^{\infty} J_n = [1, 2]$.

Problem 5. Prove that $\left(\lim \left(\frac{5n + 3}{n + 7} \right) = 5 \right)$.

Problem 6. Find the limit of $\sqrt{(1 + \frac{1}{n^2 + 5}).(\frac{2n + 1}{n^2 + 7} + 2)}$.