

MAT 319 Spring 2015
Review for Final Examination

The Final is cumulative.

Review of material covered in Midterm 1. There will be questions in which students are asked to give one of the following definitions:

- x is an algebraic number. (Ross, 2.1)
- definitions of upper bound, lower bound and bounded (Ross, 4.2)
- $\inf S$ and $\sup S$ for any bounded set S of real numbers.
- be able to state the Completeness Axiom (Ross, 4.4)

and to prove and be able to apply one or more of the following theorems:

- Rational Zeros Theorem (Ross, 2.2)
- the Archimedean Property (follows from the Completeness Axiom). (Ross, 4.6)

Also,

- Understand why “Proof by induction” works (using Axiom 5; Ross, §1), and be able to use this method of proof.
- Understand and be able to apply the Triangle Inequality. (Figure 3.2)

Review for Midterm II, as before:

This midterm will cover material in Ross, through §16 (except §§12 and 13). There will be questions in which students are asked to give one of the following definitions:

- The sequence (s_n) converges to s . (Ross, Definition 7.1)
- The sequence (s_n) is a Cauchy sequence. (Ross, Def. 10.8)
- The series $\sum_0^\infty a_n$ converges. (Ross, §14.2)
- State the Ratio Test. (Notes on series and Ratio Test)

and to prove one of the following theorems:

- The limit of a sequence is unique. (Ross, end of §7)
- Every monotone, bounded sequence converges. (Ross, 10.2)
- If a series $\sum a_n$ converges, then $\lim a_n = 0$. (Notes on series and Ratio Test)
- Geometric series: $\sum_{k=0}^\infty ar^k$ converges to $a/(1-r)$ if $|r| < 1$ and diverges otherwise. (Ross, §14, Example 1)
- $e = \sum_{k=0}^\infty \frac{1}{k!}$ is irrational. (Ross, §16, Example 6)

- An infinite decimal corresponds to a rational number if and only if it repeats. (Ross, 16.5)

Other problems will include applications of the Comparison Test, the Ratio Test, the Integral Test, and the Alternating Series Test to various specific series. Review Exercises 14.1-4 and 15.1-4.

Review of material since Midterm II

Students are not responsible for material in §§19-22.

There will be questions in which students are asked to give one of the following definitions:

- The function f is continuous at the point x_0 . (Ross, 17.1)
- The function f is continuous on a set S . (Ross, 17.1)
- The function f is bounded on the set S . (Ross, start of §18)

and to prove one of the following theorems:

- The “sequential” definition of continuity at x_0 is equivalent to the “ $\epsilon - \delta$ ” definition (Ross, 17.2)
- If f is continuous at x_0 and $f(x_0)$ is in the domain of g , and g is continuous at $f(x_0)$, then the composed function $g \circ f$ is continuous at x_0 . (Ross, 17.5)
- If f is continuous on $[a, b]$, then f is bounded.
- If f is continuous on $[a, b]$, let $M = \sup\{f(x), a \leq x \leq b\}$ and $m = \inf\{f(x), a \leq x \leq b\}$; then there exist x_1, x_2 both $\in [a, b]$ such that $f(x_1) = M, f(x_2) = m$. (Ross, 18.1)
- Intermediate Value Theorem (Ross, 18.2)

For power series, students are responsible for the material in “Notes on Power Series” linked to the Syllabus page. In particular

- Be able to apply the Ratio Test as in Prop. 1 to determine the radius of convergence of a given series.
- Understand the concept of uniform convergence and be able to show by examples how it differs from ordinary (pointwise) convergence. (Def. on p. 1 and discussion afterwards).
- Be able to prove that a uniform limit of continuous functions is continuous (the $\epsilon/3$ proof). Be able to show with an example why “uniform” is necessary.
- Be able to apply Propositions 6, 7, 8, 9 to examples of infinite series.