Section 2.1

1b) The four symmetries, and the associated permutations of the vertices, are as follows:

The identity map ($A \to A$, $B \to B$, $C \to C$, $D \to D$);

rotation through 180 degrees ($A \to C$, $B \to D$, $C \to A$, $D \to B$);

reflection through the horizontal axis ($A \to D$, $B \to C$, $C \to B$, $D \to A$);

reflection through the vertical axis ($A \to B$, $B \to A$, $C \to D$, $D \to C$).

2) a) $h_2 \circ h_3 = h_4$

b) $h_3 \circ h_7 = h_8$

c) $h_8 \circ h_1 = h_8$

d) $(h_3 \circ h_4) \circ h_2 = h_2 \circ h_2 = h_3$

e) $h_3 \circ (h_4 \circ h_2) = h_3 \circ h_1 = h_3$

3)

$\circ$	f_1	f_2	f_3	f_4	f_5	f_6
f_1	f_1	f_2	f_3	f_4	f_5	f_6
f_2	f_2	f_3	f_1	f_6	f_4	f_5
f_3	f_3	f_1	f_2	f_5	f_6	f_4
f_4	f_4	f_5	f_6	f_1	f_2	f_3
f_5	f_5	f_6	f_4	f_3	f_1	f_2
f_6	f_6	f_4	f_5	f_2	f_3	f_1

6) A scalene triangle has only 1 symmetry, the identity. The reason is that any symmetry must permute the vertices of the triangle. Since all of the sides are different lengths, the angles must also be all distinct. Therefore, since symmetries preserve angles, each vertex must be mapped to itself.

10) Rotation by any angle around the center of the circle is a symmetry of the circle. The rotations are parameterized by angle θ, where $0 \leq \theta < 2\pi$ is a real number. These rotations are all distinct, and so the circle has an infinite number of symmetries.

Another argument uses reflections. Each reflection over a line that passes through the center of the circle is a symmetry of the circle. There are an infinite number of such lines, the vertical line and one line for any slope m in $\mathbb{R}$. Each reflection fixes exactly two points, the two points where the line of reflection intersects the circle, and so each distinct line corresponds to a different reflection.

13) $(f \circ g)(x) = (x^2) + 1 = x^2 + 1$. $(g \circ f)(x) = (x + 1)^2 = x^2 + 2x + 1$. Thus, for any $x \neq 0$, $(g \circ f)(x) \neq (f \circ g)(x)$ because $2x \neq 0$. Therefore, $g \circ f \neq f \circ g$.

Section 2.2

2b) For instance, $(b \circ d) \circ b = c \circ b = a$ while $b \circ (d \circ b) = b \circ d = c$.

3 b) There is no identity element.

c) d is an identity element.

5) a) It is commutative.

b) It is not commutative. For instance, $d \circ b = a$ while $b \circ d = b$.

7) a) $(2.6 \times 1.3) \times 3.7 = 3.4 \times 3.7 = 12.6$
$2.6 \times (1.3 \times 3.7) = 2.6 \times 4.8 = 12.5$

b) $(2.6 \times 1.3) \times 3.7 = 3.3 \times 3.7 = 12.2$
$2.6 \times (1.3 \times 3.7) = 2.6 \times 4.8 = 12.4$

8) First, 1 has an inverse, namely 1; -1 has an inverse, namely -1. 0 has no inverse; $0 \cdot x = 0$ for all x, so there is no x such that $0 \cdot x = 1$. For any other $x \in \mathbb{Z}$, x has an inverse in the rationals, namely $\frac{1}{x}$, but this is not an integer unless $x = 1$ or -1.

Section 2.3

1) After 52 minutes from 10:26, then minute counter will show 18 minutes.

2) a) $3 + 5 = 2$

b) $0 + 4 = 4$

c) $(2 + 3) + 2 = 5 + 2 = 1$

d) $-4 = 2$

3) $(3 + 5) + 2 = 2 + 2 = 4$ and $3 + (5 + 2) = 3 + 1 = 4$.

6) a)

$\cdot$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

b) $(\mathbb{Z}_4, \cdot)$ is not a group because 0 and 2 do not have inverses.

8) $\cdot$ is not a law of composition on $\{1, 2, 3, 4, 5\}$ in $\mathbb{Z}_6$ because $2 \cdot 3 = 0$, and 0 is not in the set. The set is not closed under the operation.

14) Since $(H, \cdot)$ is not commutative, there are two elements of H, call them h_1 and h_2, such that $h_1 \cdot h_2 \neq h_2 \cdot h_1$. Let g_1 and g_2 be two elements of G. Then

$$(g_1, h_1) * (g_2, h_2) = (g_1 \circ g_2, h_1 \circ h_2)$$

and

$$(g_2, h_2) * (g_1, h_1) = (g_2 \circ g_1, h_2 \circ h_1)$$

Since the second component of the two right-hand-sides are not equal, the left-hand-sides are not equal as well. Therefore, $(G \times H, *)$ is not commutative.

15) An isomorphism preserves all properties of the group, such as commutativity. Since $(\mathbb{Z}_8, +)$ is commutative and the symmetries of the square are not, the two groups cannot possibly be isomorphic.

17) The two groups are not isomorphic. Take any element x of $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$. We know that $x + x = (0, 0, 0)$. Therefore, there is no element that can correspond to 1 since $1 + 1 = 2 \neq 0$.