Section 1.3

1. a. $(1100) + (0110) = (1010)$

c. $(00010) + (10110) + (11011) = (01111)$

4c.

+	(00000)	(01110)	(10111)	(11001)
(00000)	(00000)	(01110)	(10111)	(11001)
(01110)	(01110)	(00000)	(11001)	(10110)
(10111)	(10111)	(11001)	(00000)	(01110)
(11001)	(11001)	(10111)	(01110)	(00000)

This may be the set of code words for a group code since the set is closed under addition.

5. The set $\{(0011), (1001), (0100), (1101)\}$ cannot form a group code because it does not include (0000).

7. The code words of Example 1.28 can be used for a single-error correcting code using the minimum-likelihood decoding scheme because $d = 3$; Theorem 1.3 applies.

10. Take, for instance, the code words $(000), (110), (111)$. Then d', the minimum weight of non-zero code words, is 2. However, $H((110), (111)) = 1$, so $d = 1$.

Section 1.4

2b. $(1, -2, 4, 0) \cdot (0, -3, -2, 1) = 1 \cdot 0 + (-2) \cdot (-3) + 4 \cdot (-2) + 0 \cdot 1 = 6 - 8 = -2$

3. b. $(10001) \cdot (11100) = 1 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 + 0 \cdot 0 + 1 \cdot 0 = 1$

d. $(110000) \cdot (000011) = 1 \cdot 0 + 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 1 + 0 \cdot 1 = 0$

4. b.

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

d.

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

6. $B + C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$. So

$$(B+C) \cdot A = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$C \cdot A = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

Therefore, $B \cdot A + C \cdot A = \begin{pmatrix} 1 & 1 \end{pmatrix}$.

9. The weights of the nonzero code words are as follows:

(0100101)	3
(1000011)	3
(1100110)	4
(0001111)	4
(0101010)	3
(1001100)	3
(1101001)	4
(0010110)	3
(0110011)	4
(1010101)	4
(1110000)	3

$$\begin{array}{lc} (0011001) & 3 \\ (0111100) & 4 \\ (1011010) & 4 \\ (1111111) & 7 \end{array}$$

Since this is a group code, the d is the smallest weight: 3.

10b.

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{pmatrix} = \begin{pmatrix} c_2 + c_3 \\ c_1 + c_4 \\ c_1 + c_2 + c_5 \end{pmatrix}$$

Therefore, c_3 is a parity check for c_2, c_4 is a parity check for c_1, and c_5 is a parity check for c_1 and c_2. The null space consists of $\{(00000), (10011), (01101), (11110)\}$. The smallest non-zero weight is 3, and since this is a group code, $d = 3$.

17. a.

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} c_1 + c_2 + c_3 \\ c_2 + c_4 \end{pmatrix}$$

Thus c_3 is a parity check for c_1 and c_2, and c_4 is a parity check for just c_2. The null space is $\{(0000), (1010), (0111), (1101)\}$.

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} c_2 + c_4 \\ c_1 + c_3 + c_4 \end{pmatrix}$$

Here, c_2 is a parity check for c_4, and c_1 is a parity check for c_3 and c_4. The null space is $\{(0000), (1101), (1010), (0111)\}$. The null spaces do indeed coincide.

b. Each row of a matrix yields a specific equation that determines the null space. Interchanging rows only interchanges these equations; it does not change the equations. Hence, it does not change the null space.