

**MAT 312/AMS 351 – Fall 2010**  
**Homework 5**

1. Explain in your own words why, if  $n$  is a prime, a linear congruence equation  $ax \equiv_n b$  always has a solution (i.e. that given any integers  $a$  and  $b$ , there exists an integer  $x$  such that  $ax - b$  is a multiple of  $n$ ), and that any two solutions are the same modulo  $n$ .
2. Solve  $3x \equiv_{19} 16$ . *Not* by trial-and-error please.
3. Solve  $5x \equiv_{14} 12$  (14 not prime, but  $(5,14) = 1$  sufficient).
4. Review the procedure for the case  $(a, n) = d \neq 1$ .
5. Explain why  $6x \equiv_{21} 2$  has no solutions.
6. Show that  $6x \equiv_{21} 9$  if and only if  $2x \equiv_7 3$ .
7. Solve  $2x \equiv_7 3$ . Let  $x_0$  be the unique solution.
8. Check that  $x_0$ ,  $x_1 = x_0 + 1 \cdot 7$  and  $x_2 = x_0 + 2 \cdot 7$  are all *different* solutions of  $6x \equiv_{21} 9$ . Why does this not work for  $x_0 + 3 \cdot 7$ ?
9. Find the seven solutions of  $14x \equiv_{35} 21$ .