

MY NAME IS:

Problem	1	2	3	4	5	Total
Score						

MAT 312
Applied Algebra
Midterm 1
October 5, 2010

NO BOOKS OR NOTES MAY BE CONSULTED DURING THIS TEST.

Explain your answers carefully.

Total score = 100.

- (15 points) Prove that $\sqrt{17}$ is not rational, by showing that it cannot be written as p/q with p and q integers, $(p, q) = 1$. You may use the fact that every integer can be uniquely (up to order) written as a product of primes.
- (20 points) Observe that $1 = 1^2$, $1 + 3 = 2^2$, $1 + 3 + 5 = 3^2$. Use induction to prove that the sum of the first n odd numbers is equal to n^2 , i.e.

$$1 + 3 + \cdots + (2n - 1) = n^2.$$

- (15 points) Observe that $1 - 1 = 0$, $1 - 2 + 1 = 0$, $1 - 3 + 3 - 1 = 0$, $1 - 4 + 6 - 4 + 1 = 0$. Prove that the alternating sum of the elements in the n -th row of Pascal's triangle is equal to 0, i.e.

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots \pm \binom{n}{n} = 0.$$

- (a) (10 points) Find the greatest common denominator d of 731 and 645.
(b) (10 points) Express d in the form $731j + 645k$, with j and k integers.
(c) (10 points) Give prime factorizations for 81, 82, 83.
- (a) (10 points) Calculate the multiplicative inverse of 19 *modulo* 21.
(b) (10 points) Explain why 14 does not have a multiplicative inverse *modulo* 21.

END OF EXAMINATION