# MAT125 Fall 2007 Review for Final Examination

*The Final Exam is cumulative, so the review sheets for Midterm 1 and for Midterm 2 should be used along with the following.*

**4.1** Work through Examples 1, 2, 3, 4, 5 understanding how they reflect the "Strategy" explained on p.265. Exercises 4, 9, 18 and as many others as you can fit in.

**4.2** Understand the definitions (Box 1, p.269) of "$f$ has an absolute maximum at $c$" and "$f(c)$ is the maximum value of $f$" (and for "minimum" also). Understand the definition (Box 2, p270) of "$f$ has a local maximum at $c$," etc. and study Example 4 carefully. Understand what Fermat's Theorem says (Box 4, p.272) and the situations of Figure 9 (you can have $f'(c) = 0$ without $c$ being a local max or min) and Figure 10 ($f$ might not have a derivative at the point $c$ where $f(c)$ is minimum or maximum) . Be able to implement the "Closed Interval Method" (Box, p.273): Example 7, Exercises 5, 19, 25, 39.

**4.3** Understand that if $f'(x) > 0$ on an interval, then $f$ is "sloping up" and therefore increasing on that interval; and that if $f'(x) < 0$ on an interval, then $f$ is "sloping down" and therefore decreasing on that interval (Box, p.280), Example 2, Exercises 7a, 8a, 9a. Understand the relation between $f''$ and the concavity of the graph of $f$ (Box, p.282) and remember Paul Kumpel's mnemonograms:

$$\begin{array}{ccc} +\;+ & \qquad & -\;- \\ \smile & & \frown \end{array}$$

Understand what an *inflection point* is (just below Figure 5 on p.282). Exercises 7c, 8c, 9c. The Second Derivative Test (Box, p.282) will be very useful when you get to optimization problems, in section 4.6. Be able to put slope and concavity information together with information about asymptotes (from section 2.5) to sketch graphs of complicated functions. Example 6, Exercises 29, 31.

**4.5** Understand *when* you can apply L'Hôpital's rule: BOTH numerator and denominator must have limit 0, or BOTH numerator and denominator must have limit $\infty$ or $-\infty$ (Box, p.298). Example 1. Be prepared to apply the rule more than once, Example 2. Exercises 5, 9, 17. Be able to rewrite an *indeterminate product* (p.300, i.e. where one factor goes to 0 and the other goes to $\infty$) as a quotient suitable for L'Hôpital's rule. Example 6, Exercises 25, 26. And *indeterminate differences* Example 7, Exercises 31, 33.

**4.6** Optimization problems are difficult because (as in related rates problems) you have to set up the notation and the mathematical context. Understand and practice applying the "Six-step process" from p.306. Steps 4 and 5 are crucial: this is where you make the word problem into a "find the maximum" (or minimum) problem. Work through Examples 1, 2, 3, 4, 5 *with the six steps in mind*, so you learn how to apply them yourself. Exercises 15, 19, 22 and as many others as you have time for. Do odd-numbered ones so you can check your answers in the back of the book.

**4.7** Understand the concept of *marginal cost* of production: if $C(x)$ is the total cost of making $x$ items, then the derivative $C'(x)$ (which is approximately the difference quotient $\frac{C(x+1)-C(x)}{1}$, i.e. the cost of making the next item) is by definition the marginal cost when the production rate is $x$. Understand that when the average cost $\frac{C(x)}{x}$ is minimum, the marginal cost equals the average cost. (Box, p.317 and the explanation just below the box). Be able to use this understanding to locate the minumum average cost by solving $C'(x) = \frac{C(x)}{x}$. Example 1, Exercise 1, 7ac, 8ac. Understand the *demand function* $p(x)$: it's the most the manufacturer can charge per item and still sell $x$ items (if the price were higher, sales would go down), bottom of p. 318. And understand that then the *revenue* $R(x)$ is the product of the price per item times the number of items sold: $R(x) = xp(x)$ (also p.318, bottom), and that the *profit* is revenue minus cost: $P(x) = R(x)-C(x)$ (p.319, top). These are all definitions. Understand that for production of $x$ items to maximize profit, you must have $P'(x) = 0$, so $R'(x) = C'(x)$, and be able to use this equation to calculate the optimal production level $x$ given the functions $C$ and $p$. Example 2, Example 3 (here you calculate $p(x)$ from data). Exercises 15, 17, 18.

Use the Chapter Reviews for further reviewing.

December 4, 2007