MAT125 Spring 2011 Review for Midterm II

2.3 Understand that limits interchange with the arithmetic operations $+, \times, -, /$, *except when they lead to division by zero* (Example 1, Exercises 2,3,7). In particular since $\lim_{x\to a} c = c$ (here c is a constant function) and $\lim_{x\to a} x = a$ (be sure you understand what this means), the limit at $x = a$ of a polynomial function of x is the value of that function at x (Example 2a). Because of possible zero-denominators, this does not always work for quotients of polynomials (Examples 2b, 3, Exercises 12, 13, 14). Be able to "rationalize" (Example 6, Exercise 18,21) to calculate limits involving radicals.

2.4 Know the definition of "f continuous at a" (Box, p.113) (Exercises 3a,13,14). Also "continuous from the right" and "from the left" (Box 2, p.115) (Exercise 3b). Understand that polynomials are continuous everywhere, and that rational functions are continuous *wherever they are defined* (Theorem 5, p.116) (Example 5). Be able to apply the Theorem on compositions (Theorem 8 p.119) (Example 8). Understand how to use the Intermediate Value Theorem to calculate roots of equations by systematic trial and error (Example 9, Exercises 41, 48).

2.5 Understand how $\lim_{x\to a} f(x) = \infty$, etc., give a *vertical asymptote* at a (Box 2 p.125, Example 1 p.126). Exercises 3, 7. Understand how $\lim_{x\to\infty} f(x) = L$, etc., give as *horizontal asymptote* the line $y = L$ (Box 5 p.128, Examples 3,4 p.129). Exercises 3, 7.

Be able to calculate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for the special case where $f(x) = p_1(x)/p_2(x)$ is the quotient of two polynomials, p_1, p_2. Use the "divide by the highest power of x in the denominator" method (explained in Example 5 p.130, Example 10 p.132). Exercises 22, 39, 40.

2.6 Understand that the slope of the tangent line to the graph of f at the point $(a, f(a))$ is the limit of the slopes of secant lines (Figure 1, Box 1, p.135). Be comfortable with both notations: $x \to a$ and $h \to 0$, where $h = x - a$. (Compare Box 1 and Box 2, p.136). *If this limit exists,* it is called the derivative of f at a. Be able to use this slope and the point-slope method to give the equation of the tangent line: Examples 1, 2 p.136, Example 5 p. 139. Exercises 7, 10. Understand also that instantaneous velocity at $t = a$ is the limit of average velocities over smaller and smaller time periods beginning or ending with a. Box 3 p.137. Example 3. Exercises 13, 16ab.

2.7 Basic concept: the derivative of f at x (Box 2 p.146) as a new function of x, written f'. So $f'(a)$ is the slope of the line tangent to the graph of f at the point $(a, f(a))$. Be able to use the graph of f to make a rough sketch of the graph of f' as in Example 1 p.146. (Box, Example 2, p.149). Be able to calculate $f'(x)$ from the definition in simple cases (Examples 3, 4, 5 pp.148-149), Exercises 19-22. Understand why $f(x) = |x|$ does not have a derivative at $x = 0$ (Example 6 p.150). Understand how to calculate the second derivative $f''(x)$ (Example 7 p.153) and its interpretation in terms of acceleration (Example 8 p.154; Exercise 42, 43, 48a).

2.8 Be able to tell by examining f' where f is increasing and where it is decreasing (Box, p.158; Example 1, Exercises 1,2). Understand the definition of *local maximum* and *local minimum* (in text, p.159). Be able to tell from f'' where the graph of f is concave upward and where it is concave downward (Box, p.159; Example 2, Exercises 27, 28). Be able to

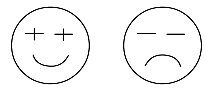

Figure 1: The Kumpel mnemonic for the sign of f'' and concavity.

sketch a possible graph for f, given information about f' and f'' (Example 3 p.160, Exercises 21, 22).

3.1 Know the elementary differentiation rules: $\frac{d}{dx}(c) = 0$ and $\frac{d}{dx}(x) = 1$ (Boxes, p.174) and understand what these equations mean in terms of slopes. Know the *Power Rule*: $\frac{d}{dx}(x^n) = nx^{n-1}$ (Boxes, p.175 and p.176). Be familiar with the special cases $n = \frac{1}{2}$ ($f(x) = \sqrt{x}$) and $n = -1$ ($f(x) = \frac{1}{x}$). Examples 2, 3. Be able to calculate the derivative of $rf(x) + sg(x)$ for constants r, s knowing the derivatives of f and g separately. (Boxes, pp.177-178 Examples 4, 5, 6). Exercises 11, 12, 13.

Know how to differentiate the "natural exponential function" $f(x) = e^x$ (Box, p.180; Examples 8, 9, Exercises 14, 15).

3.2 Be able to apply the product and quotient rules correctly (Box, p.184; Examples 1a, 2, 3; Exercises 3, 4, 25); (Box, p.187; Examples 5, 6; Exercises 11, 19). If you can't remember where the minus sign goes in the quotient rule, use $\frac{d}{dx}\frac{1}{x} = \frac{-1}{x^2}$ to check.

3.3 Be able to sketch the graphs of $\sin x$ and $\cos x$ to scale ($\pi = 3.14..$) and to convince yourself that $\sin' = \cos$ and that $\cos' = -\sin$. (Boxes 4, 5 and Example 1, p.193; Exercises 3, 4, 10). Remember or be able to calculate that $\tan' = \sec^2$ (Box, p. 194). Know the definitions of $\sec, \csc, \cot$ so you can calculate their derivatives from the quotient rule. [This is a good place to review your trigonometric identities, especially $\sin^2 + \cos^2 = 1$ and $\tan^2 + 1 = \sec^2$.] Example 2, Exercises 8, 14.

Use the Chapter Reviews for further reviewing.
[In Chapter 3, only
Concepts 1 abcdef, 2 abfghijk, 3abc
True-False 1, 2, 6, 11, 12
Exercises 1, 3, 6, 9, 38, 51, 52]

March 17, 2011