MAT 303 MIDTERM

Write full solutions to these problems; final answers alone are worth nothing.

You may use static online resources (e.g. online books), but you are not allowed to search for anything specific to these problems (e.g. on WolframAlpha) or access any type of Q&A site (Chegg, StackExchange, Quora, etc.).

PART I: ODES

NOTE: In *Part I* of the exam, we alternate between having t as the independent variable, and giving an equation involving x and y.
In the former case, solve for y as a function of t, and in the latter case, leave the solution as a relation on x and y.

1. (a) $y' + 3y = t + e^{-2t}$
   (b) $y' + \frac{1}{t}y = 3\cos(2t)$
   (c) $y' + y^2 \sin(t) = 0$
   (d) $y' = \frac{x^2 + 3y^2}{2xy}$
2. (a) $(3x + 2y)dx + \left(x + \frac{6y^2}{x}\right) dy = 0$
   (b) $-y^2 \sin(xy)dx + (-xy\sin(xy) + 2\cos(xy) + 3y)\, dy = 0$
3. Find a particular $g(t)$ satisfying $W(f, g) = t^2 e^t$ with $f(t) = t$
4. Verify that y_1 and y_2 are solutions to the corresponding homogeneous equation and find the general solution of the inhomogeneous equation:
$$t^2 y'' - t(t+2)y' + (t+2)y = 2t^3$$
$$y_1(t) = t, \quad y_2(t) = te^t$$
5. Given that $y_1(t) = 1 + t$ is a solution of the corresponding homogeneous equation, find the general solution of the inhomogeneous equation:
$$ty'' - (1+t)y' + y = t^2 e^{2t}$$
6. Find the general solution of:
$$y''' + 2y'' - y' - 2y = \frac{12e^t}{e^{2t} + 1}$$

PART II: ODE SYSTEMS

NOTE: Solve for x, y, z as functions of t. All solutions must be real.

7. (a)

$$\begin{cases} x' &= x + 3y \\ y' &= -2x - 3y \end{cases}$$

(b)

$$\begin{cases} x' &= x + y + \dfrac{e^{4t}}{e^{2t} + 1} \\ y' &= -2x + 4y \end{cases}$$

(c)

$$\begin{cases} x' &= -2y + \tan(t) \\ y' &= 2x \end{cases}$$

8. (a)

$$\begin{cases} x' &= x + y + 2z \\ y' &= x + 2y + z \\ z' &= 2x + y + z \end{cases}$$

(b)

$$\begin{cases} x' &= -3x - 13y - 4z \\ y' &= 7x + 35y + 11z \\ z' &= -22x - 94y - 29z \end{cases}$$

(c)

$$\begin{cases} x' &= 4x \\ y' &= 4y + z \\ z' &= 2x + 4z \end{cases}$$

(d)

$$\begin{cases} x' &= -\dfrac{1}{2}x + \dfrac{1}{2}y - \dfrac{1}{2}z + 1 \\ y' &= -x - 2y + z + t \\ z' &= \dfrac{1}{2}x + \dfrac{1}{2}y - \dfrac{3}{2}z + 11e^{-3t} \end{cases}$$