

MAT 303 FINAL

Write full solutions to these problems; final answers alone are worth nothing.

You may use static online resources (e.g. online books), but you are not allowed to search for anything specific to these problems (e.g. on WolframAlpha) or access any type of Q&A site (Chegg, StackExchange, Quora, etc.).

PART I: CORE MATERIAL

NOTE: I consider the *Core Material* section of the exam to be the more important one, and you should solve most of these problems prior to continuing.

1. (a) $y' = \frac{3t}{y + t^2y}$
(b) $y' = (1 + y^2) \tan(2t)$
(c) $t^2y' + 2ty - y^3 = 0$
(d) $y' = \frac{x + 3y}{x - y}$
(e) $(-y \sin(x) + y^3 \cos(x)) dx + (3 \cos(x) + 5y^2 \sin(x)) dy = 0$
(f) $(2x \sin(y) + 1) dx + (4x^2 \cos(y) + 3x \cot(y) + 5 \sin(2y)) dy = 0$
2. (a) $y'' - y = \frac{4}{e^{2t} + 1}$
(b) $y'' + 8y' + 25y = 36e^{-4t} - 160 \cos(t)$
(c) $t^2y'' + 4ty' + 2y = \frac{4t}{t^2 + 1}$
(d) $y''' - 2y'' - y' + 2y = \frac{12e^{2t}}{e^t + 1}$
3. (a)
$$\begin{cases} x' &= x - 2y + \sec(t) \\ y' &= x - y \end{cases}$$

(b)
$$\begin{cases} x' &= 5x - y + \frac{4e^{4t}}{t^2 + 1} \\ y' &= x + 3y \end{cases}$$

(c)
$$\begin{cases} x' &= -x + z \\ y' &= y + 2z \\ z' &= 2y + 4z \end{cases}$$

(d)

$$\begin{cases} x' &= 2x \\ y' &= x - y - z \\ z' &= -2x + y - z \end{cases}$$

PART II: ADDITIONAL TOPICS

4. Solve the following by way of inverse operators:

(a) $y''' - y'' + y' - y = 2t^4$

(b) $y''' + 3y'' + 3y' + y = e^{-t}(2 - t^2)$

5. Solve the following by way of Laplace transforms:

(a) $3y''' + 5y'' + y' - y = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = -1$

(b) $y''' - 2y'' + y' = 2e^t + 2t$, $y(0) = y'(0) = y''(0) = 0$

(c)

$$\begin{cases} x'' - 4x + y' = 0 \\ -4x' + y'' + 2y = 0 \\ x(0) = 0, \quad x'(0) = 1 \\ y(0) = -1, \quad y'(0) = 2 \end{cases}$$

(d)

$$\begin{cases} x' + 4x + y' = 1 \\ x' - 2x + y = t^2 \\ x(0) = 2 \\ y(0) = -1 \end{cases}$$

(e)

$$\begin{cases} x'' - y' = 1 - t \\ x' - x + 2y' = 4e^t \\ x(0) = 0, \quad x'(0) = 1 \\ y(0) = 0 \end{cases}$$