

MAT 126 MIDTERM

Write full solutions to these problems; final answers alone are worth nothing.

You may use static online resources (e.g. online books), but you are not allowed to search for anything specific to these problems or access any type of Q&A site (Chegg, StackExchange, Quora, etc.).

REMINDER: Recall that we use \log to denote the natural logarithm (base e).

1. To be solved by algebraic manipulation and inspection:

(a) $\int (x^3 + 2)^2 dx$

(b) $\int \frac{2x^4 - 3x^3 + 3x^2 - 3x + 1}{x^2 + 1} dx$

(c) $\int \frac{\cos(x)}{\sec(x) - \tan(x)} dx$

2. To be solved by substitution:

(a) $\int e^{e^x} e^x dx$

(b) $\int \frac{x^2}{\sqrt{1 - x^6}} dx$

3. To be solved by parts:

(a) $\int x^2 \log(x) dx$

(b) $\int x^2 \sin(x) dx$

4. To be solved by partial fractions:

(a) $\int \frac{x + 1}{x^3 + x^2 - 6x} dx$

(b) $\int \frac{x^3 + 4x^2 + 10}{(x - 1)^2(x^2 + 4)} dx$

5. To be solved using an assortment of trig integral techniques:

(a) $\int \sin^4(x) dx$

(b) $\int \frac{\sqrt{9 + x^2}}{x^2} dx$

$$(c) \int \log(x + \sqrt{x^2 - 1}) \, dx$$

$$(d) \int \frac{1}{3 + 2 \cos(x) + \sin(x)} \, dx$$

Note: *The inverse trig function in d) cannot be simplified.*

6. Find the derivatives of the following functions:

$$(a) F(x) = \int_{-x}^{x^2} \log(\cos(t)) \, dt$$

$$(b) F(x) = \int_0^{x^2} \frac{1}{1+t^2} t^5 \, dt$$

7. Using Riemann sums, show that the function $f(x) = x$ is integrable on $[0, 1]$

and determine $\int_0^1 f(x) \, dx$.