MAT 126: PROBLEM SET 5

1. Express the following polar equations in Cartesian form:
   (a) $r = 2a\sin(\theta)$
   (b) $r = a\csc(\theta)$
2. Express the following Cartesian equations in polar form:
   (a) $y = \dfrac{1}{x}$
   (b) $(x-1)^2 + (y-1)^2 = 2$
3. Sketch the following curves and find the area that they enclose:
   (a) $r = \theta^2$ for $-\pi \leqslant \theta \leqslant \pi$
   (b) $r = a + b\sin(\theta)$, where $a > b > 0$, for $0 \leqslant \theta \leqslant 2\pi$
4. Find the area inside both of the curve $r = \sin(2\theta)$ for $0 \leqslant \theta \leqslant \pi/2$ and the circle $r = \cos(\theta)$. Accompany your solutions with a sketch.
   Hint: We found the Cartesian equation of this circle in class.
5. Evaluate the following improper integrals or show that they diverge:
   (a) $\displaystyle\int_0^{\frac{\pi}{2}} \tan(x)\,dx$
   (b) $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\,dx$
   (c) $\displaystyle\int_0^{\infty} \frac{\arctan(x)}{1+x^2}\,dx$
6. Using comparison tests, establish whether or not these integrals converge:
   (a) $\displaystyle\int_0^{\infty} \frac{1}{e^x + e^{-x}}\,dx$
   (b) $\displaystyle\int_0^{\infty} \frac{4^x}{2^x + 3^x}\,dx$
7. *Gabriel's Horn* is the volume obtained by revolving about the x-axis the region lying between by the x-axis and the graph of $y = 1/x$ over the interval $[1, \infty)$. Determine the volume of Gabriel's Horn.

   Bonus: Using methods not covered in class, it is possible to show that the Horn has infinite surface area. Intuitively, we can think of surface area as measuring the amount of paint required to paint a surface. However, if we fill Gabriel's Horn with a finite volume of paint, then is it not the case that the surface will get fully coated? Explain this apparent contradiction.