MAT 126: PROBLEM SET 4

NOTE: *Some of the questions on this assignment should provide practice with performing substitutions in definite integrals and changing bounds.*

1. Find the area of the region fully enclosed by the graphs of the functions $f(x) = -x^2 + 3x$ and $g(x) = 2x^3 - x^2 - 5x$.

2. Consider the region bound on three sides by the x-axis, and the graphs of the functions $y = \sqrt{x}$ and $y = \dfrac{3}{2} - \dfrac{x}{2}$.

   (a) Find its area by integrating with respect to x.

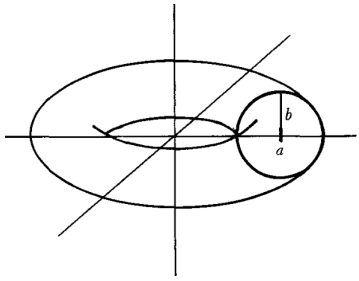
   (b) Find its area by integrating with respect to y.

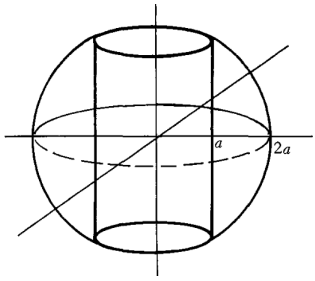
3. Consider the region bound by the curves $y = x$ and $y = x^2$. Find the volume obtained by revolving this region:

   (a) About the x-axis.

   (b) About the y-axis.

4. Assume that $a > b > 0$. The equation $(x-a)^2 + y^2 = b^2$ describes a circle of radius b centred at the point $(a, 0)$. By revolving the region enclosed by the circle around the y-axis, we obtain a solid *torus* (aka. doughnut); find its volume.





5. A cylindrical hole of radius a is bored through the centre of a sphere of radius $2a$. Find the volume of the remaining solid.

   Hint: You know the value of $\arcsin(1/2)$ via the *30-60-90* triangle.

6. Find the arc length of:

   (a) The curve $f(x) = \dfrac{1}{24}x^3 + \dfrac{2}{x}$ over $[2, 4]$.

   (b) The curve $f(x) = \dfrac{1}{2}\left(e^x + e^{-x}\right)$ over $[0, 1]$.

   Hint: Both of these problems yield perfect squares.

7. As promised, here are some integration problems to keep you fresh:

   (a) $\displaystyle\int_0^{\pi/3} \frac{1}{2-\cos(x)}\,dx$

   (b) $\displaystyle\int_{2\sqrt{2}}^{4} \frac{1}{x\sqrt{x^2-4}}\,dx$

   (c) $\displaystyle\int_0^{\pi/6} \cos^5(3x)\,dx$

   (d) $\displaystyle\int_0^{1} \frac{5x^2}{(x^2-4)(x^2+1)}\,dx$

   Hint: All of these definite integrals have nice final answers. In particular, these answers do not involve (inverse) trig functions, so be sure to reduce fully. (Recall your two special triangles and the definitions of the trig functions as ratios.)