Same deal as last time - some of these questions will be useful, others less so. Again, you will be expected to prove things in the exam, but the best method for remembering proofs is not to memorise them, but to understand the underlying concepts. Also, some of the proofs will be of new results, which will test your understanding of the material.

**Question 1.** Express the following recurring decimals as rational numbers.
(a) $2.\overline{71828}$
(b) $2.\overline{34567}$
(c) $1.\overline{2345} + 2.\overline{19}$

**Question 2.** Show that if $A$ and $B$ are finite sets, then if $A \subseteq B$ we have
$$\min B \leq \min A$$

**Question 3.** Let $X$ and $Y$ be finite sets with $|X| < |Y|$. Show there does not exist a surjection $\phi : X \to Y$.

**Question 4.** Let $X = \{x_1, x_2, x_3, x_4\}$ and $Y = \{y_1, y_2, y_3, y_4, y_5\}$.
(a) How many maps are there $f : X \to Y$?
(b) How many maps are there $f : Y \to X$?
(c) What is $|\{f \in \text{Fun}(X,Y) \mid y_4 \notin \text{im} f\}|$?

**Question 5.** Suppose we pick 17 elements from the set $\mathbb{N}_{32}$. Show that we must have picked a pair of integers whose sum is 33.

**Question 6.** Four people visit a restaurant and each choose one meal from a choice of seven on the menu.
(a) How many possible combinations are there if we record who chose which dish?
(b) How many possible combinations are there if we do not record who chose which dish?
(c) How many possible combinations are there if we record who chose which dish and each person chose a different dish from everyone else?

**Question 7.** Suppose $X \cap Y = \emptyset$. Show that the function
$$f : \bigcup_{i=0}^{k} \mathcal{P}_i(X) \times \mathcal{P}_{k-i}(Y) \to \mathcal{P}_k(X \cup Y)$$
given by $f(A, B) = A \cup B$ is a bijection. From this, deduce that
$$\binom{m+n}{k} = \sum_{i=0}^{n} \binom{m}{i} \binom{n}{k-i}.$$
Question 9.

(a) Let $a < b$ and $c < d$. Show that the map $f: [a, b] \rightarrow [c, d]$ given by

$$f(x) = \frac{(b - x)c}{b - a} + \frac{(x - a)d}{b - a}$$

is a bijection. Deduce that any two closed intervals containing more than one point have the same cardinality.

(b) Show that all intervals containing more than one point have the same cardinality. (Hint: it is not necessary to find an explicit bijection to do this).

(c) Show that all intervals containing more than one point have the same cardinality as $\mathbb{R}$.

Question 10. By considering the map $f: [0, 1) \times [0, 1) \rightarrow [0, 1)$, defined by

$$f((0.a_1a_2\ldots a_n, 0.b_1b_2\ldots b_n\ldots)) = 0.a_1b_1a_2b_2\ldots a_nb_n\ldots$$

(and using the expansion ending in recurring 0s if there is a choice) deduce that $|\mathbb{R} \times \mathbb{R}| = |\mathbb{R}|$. 