Here are some practice questions for the final. Here are some pointers.

- You are expected to know the basic definitions covered in class. This means you should give precise mathematical definitions; for example “$f$ is an injection if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$” and NOT “$f$ is an injection if no two things map to the same thing in the codomain”.
- There will be proofs in the exam. Again, these proofs should be made up of precise mathematical arguments, where each step logically follows from the previous one. You will be penalised for “hand-wavy” or unjustified arguments. Many of the proofs will be of results covered in class (but no proof will be very long, so as a freebie, I’ll tell you that you won’t have to prove the Pigeonhole principle for example).
- If you are using a result covered in class, you should explicitly state so, perhaps by summarising what the result says. If the result has a name (e.g. Pigeonhole principle), you can use that.
- The exam will be split into two parts. The first question will cover what I consider to be the “basics” of the course. This will focus on some of the simpler ideas in each section of the notes. A good performance on question 1 will indicate a grasp of the basic concepts in the course, and will be rewarded with at least a C grade.
- The latter questions will involve some of the more difficult concepts, or more complicated examples than question 1. If you want to score a high grade, you should also be able to answer these questions too.
- Even if you can’t work out a whole proof, outline your ideas. Showing you have an understanding of the concepts underlying the proof will get some credit.

Below are the questions. In general, I will be aiming these practice questions for the later questions on the exam, but some of the questions may also be helpful for question 1. For other practice, make sure you look back over the homework problems from the course.

**Question 1.** Show that
\[
\prod_{i=2}^{n} \left(1 - \frac{1}{i^2}\right) = \frac{n + 1}{2n}.
\]

**Question 2.** Show that if $f: X \to Y$ is a surjection then there exists an injection $g: Y \to X$ (you may assume the axiom of choice\(^1\)).

**Question 3.** Let $\mathcal{L}$ be the set of lines in the plane and let $f: \mathbb{R}^3 \to \mathcal{L}$ be defined so that $f(a, b, c)$ is the line with equation $ax + by = c$. Show that $f$ is a surjection but not an injection.

**Question 4.** Let $C = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ be the unit circle in the plane. Show that $|C| = |\mathbb{R}|$.

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\(^1\)If you don’t understand this comment, feel free to be able to assume that given any set $X$, you are able to pick an element $x$ from $X$. 

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Question 5. Solve the following linear diophantine equations
   (i) $9m + 18n + 45p = 93$
   (ii) $3m + 7n + 12p = 14$
   (iii) $4m + 6n + 13p = 42$

Question 6. Solve the linear congruences.
   (a) $5x \equiv 17 \pmod{123}$
   (b) $90x \equiv 18 \pmod{135}$
   (c) $490 \equiv 84 \pmod{1428}$.

Question 7. Let $L$ be the set of lines in the plane and define a relation $\sim$ on $L$ by $L_1 \sim L_2$ if and only if $L_1 \cap L_2 \neq \emptyset$. Is $\sim$ reflexive, symmetric or transitive? Is $\sim$ an equivalence relation?

Question 8. Let $L$ be the set of lines in the plane and define a relation $\sim$ on $L$ by $L_1 \sim L_2$ if and only if $L_1 \cap L_2 = \emptyset$. Is $\sim$ reflexive, symmetric or transitive? Is $\sim$ an equivalence relation?

Question 9. Let $X = \text{Fun}(\mathbb{R}, \mathbb{R})$ be the set of functions $f : \mathbb{R} \to \mathbb{R}$. Let $\sim$ be a relation on $X$ given by
   
   $$f \sim g \iff \text{there exists a bijection } h : \mathbb{R} \to \mathbb{R} \text{ such that } g = h^{-1} \circ f \circ h.$$  

   Show that $\sim$ is an equivalence relation.
   (a) What is the equivalence class of the identity function $\text{id}_\mathbb{R}$?
   (b) What is the equivalence class of the function $f$ defined by $f(x) = 0$ for all $x \in \mathbb{R}$?