Problem 1. We will prove the Generalized Binomial Theorem; that is, for any $k$ and for $|x| < 1$

$$(1 + x)^k = 1 + kx + \cdots + \frac{k(k-1)\cdots(k-(n-1))}{n!}x^n + \cdots = \sum_{n=0}^{\infty} \binom{k}{n} x^n.$$ 

(i) Show that $\binom{k-1}{n} + \binom{k-1}{n-1} = \binom{k}{n}$

(ii) Let $g(x) = \sum_{n=0}^{\infty} \binom{k}{n} x^n$ and show that $g'(x) = k \sum_{n=0}^{\infty} \binom{k-1}{n} x^n$.

(iii) Use parts (i) and (ii) to show that $(1 + x)g'(x) = kg(x)$.
(iv) Let \( f(x) = (1 + x)^{-k}g(x) \) and, by differentiating, deduce that \( f \) is a constant function.

(v) Using the fact that \( g(0) = 1 \) and part (iv), show that \( g(x) = (1 + x)^k \).

**Problem 2.** Professors Lee and Sharland are driving to the store to buy their new kite. At one point they are traveling at 12 m/s and accelerating at 2 m/s\(^2\).

(i) Write down the second-degree Taylor polynomial that approximates the distance traveled from this point after \( t \) seconds.

(ii) Use this approximation to compute how far they travel in the following 2 seconds.

(iii) Explain why this approximation does not give a good estimate for how far they travel in the next minute.