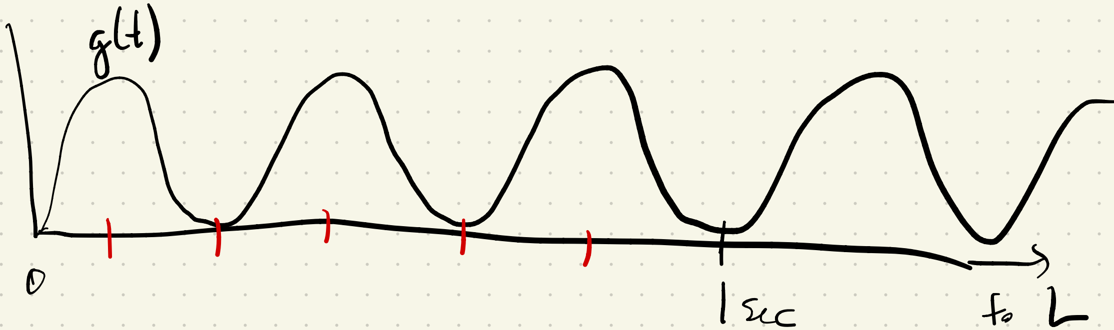
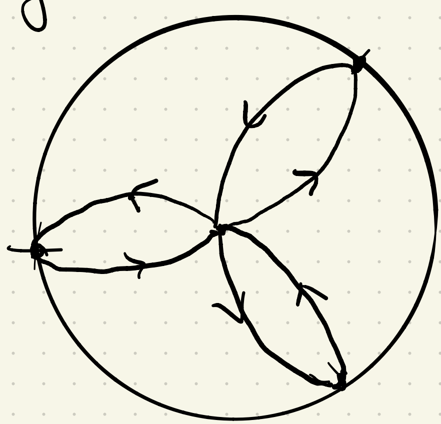
Fourier Transform

Take a wave w/ 3 beats/sec w/ time interval L.



Imagine winding this wave around in $\mathbb{C}$.

i.e. plot $g(t)\, e^{-2\pi i t}$ in $\mathbb{C}$:

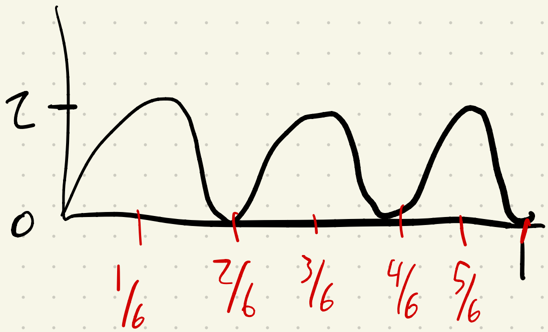


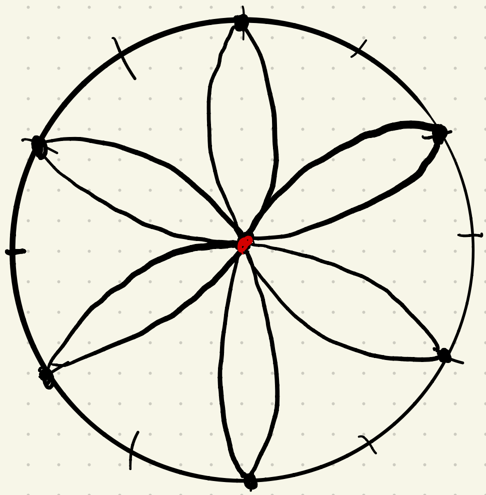
But we can add a winding parameter f (frequency) & plot

$$g(t)\, e^{-2\pi i f t}.$$

If $f = \frac{1}{2}$ we have

$$G(t) = g(t)\, e^{-\pi i t}$$





$G(0) = 0$

$G(1/6) = 2e^{-\pi i/6}$

$G(2/6) = 0$

$G(3/6) = 2e^{-\pi i/2}$

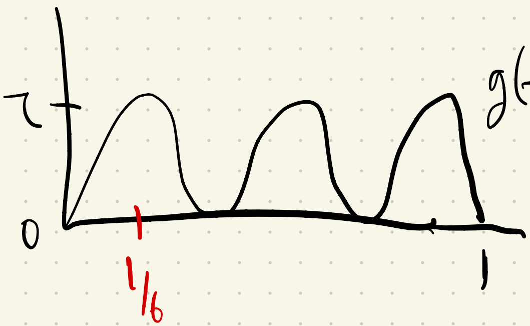
$G(4/6) = 0$

$G(5/6) = 2e^{-5\pi i/6}$

Center of mass of the image is at the origin

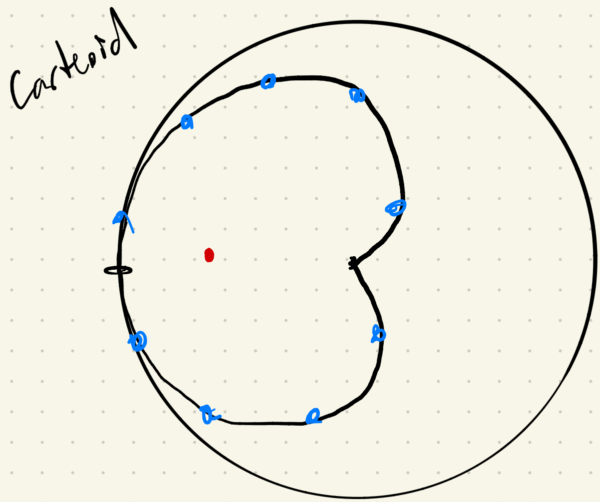
If $f = 3$, $G(t) = g(t)\, e^{-6\pi i t}$

$g(t) = 1 - \cos(6\pi t)$



$G(0) = 0$

$G(1/6) = -2$



Note that the image is mostly on the left & the center of mass is on the left, depicted as a red dot.

How to take center of mass? Can collect pts & average them. The pts are samples.

$$\frac{1}{N}\sum_{k=1}^{N} g(t_k)\, e^{-2\pi i f t_k}$$ let $N \to \infty$: $$\frac{1}{L}\int_0^L g(t)\, e^{-2\pi i \xi t}\, dt$$

write $\xi = f$

So when ξ = the frequency of $g(t)$, the center of mass is way off the origin.

If we plot the the x-val of center of mass, we get

So center of mass $= \frac{1}{L}\int_0^L g(t)e^{-2\pi i \xi t}\,dt$

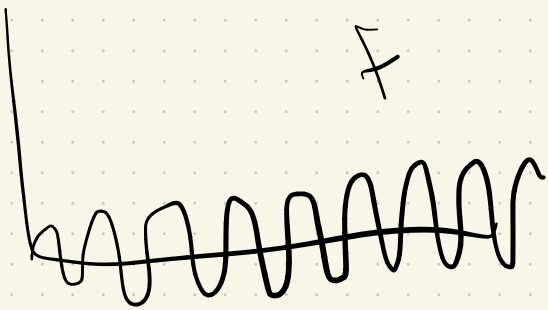
Remove the $\frac{1}{L}$; we get the Fourier transform

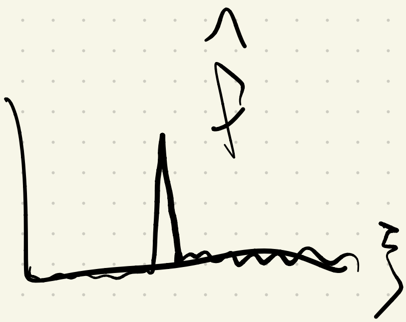
So no normalizing.

$$\hat{g}(\xi) = \int_0^L g(t)e^{-2\pi i \xi t}\,dt$$

This is how we can decompose sounds into their parts by frequency.

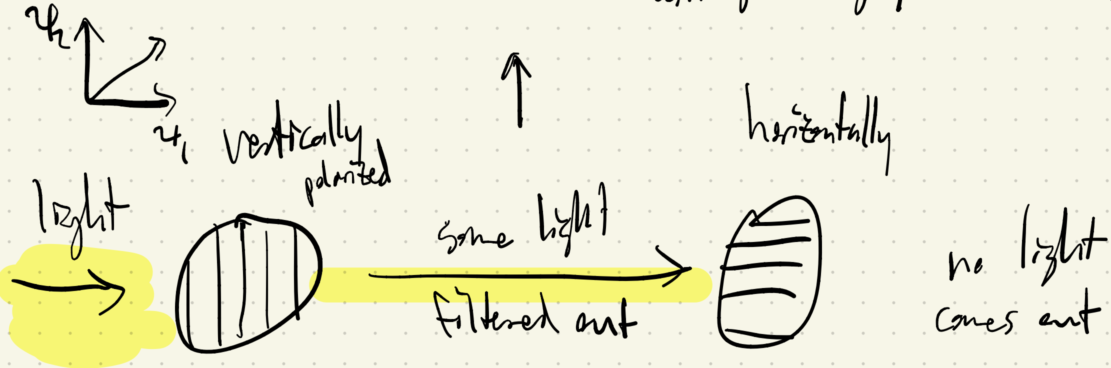
Fact: For signals w/ short span, the Fourier transform has more spread. & vice versa

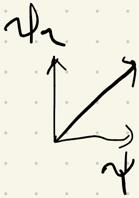




Polarized Light.

(light is absorbed in quanta)
so once wavefn collapses, a photon will go through, or not. No middle

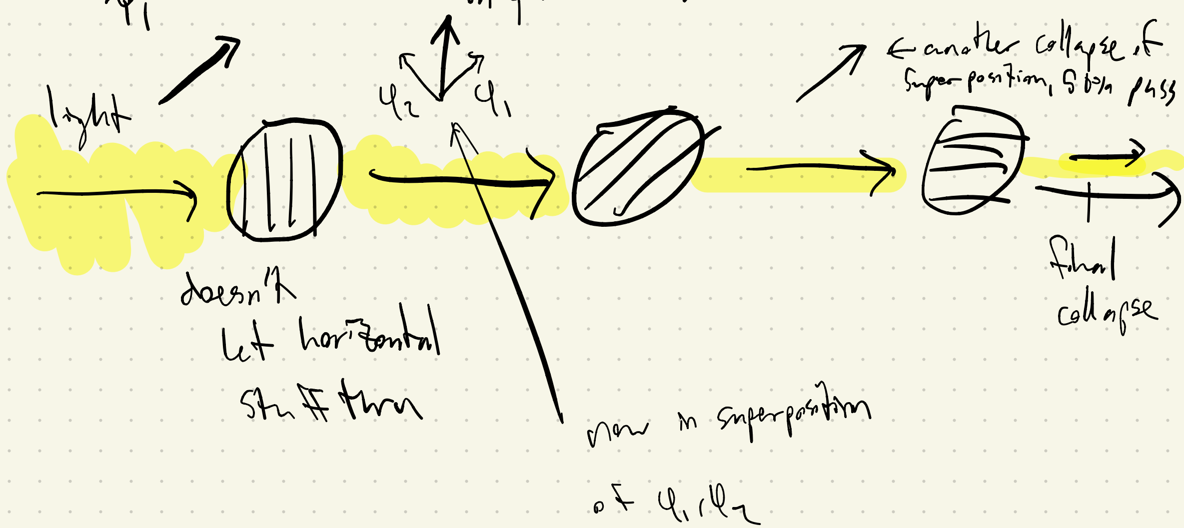




light in superposition of two

filter collapses superposition; either it is ↑ or →

only vert component passes so about 50% pass

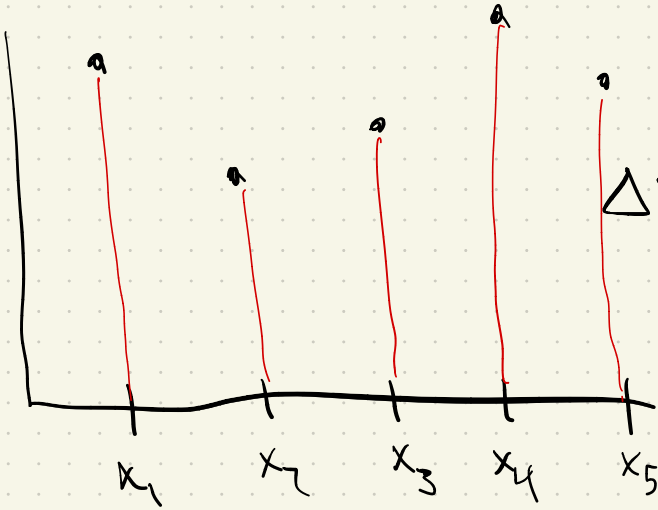


Heat Equation: (1 dim for ease)

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}.$$

Why?

Discrete setting



x_3's temp changes based on nearby pts:

$$\Delta T_3 = \frac{T_2 + T_4}{2} - T_3$$

↑ average temp of nearby pts

$$\Delta T_3 = \frac{1}{2}\left((T_2 - T_3) - (T_3 - T_4)\right)$$

$$\Delta \quad - \quad \Delta$$

^
∂_t

= difference of differences; or $\Delta\Delta \approx \partial_x^2$.