Appendix

We collect in this chapter numerical data about the constructed or studied surfaces and give references to the points in the thesis where the corresponding additional information can be found.

1. Degree 10

10.A.

Invariants
$d = 10,\ \pi = 9,\ HK = 6,\ K^2 = -3,\ \chi = 2,\ q = 0,\ N_6 = 3$

Classification
$S = S_{min}(p_1, p_2, p_3)$ non-minimal $K3$ surface
$H = H_{min} - 4E_1 - E_2 - E_3$

Syzygies of $\mathcal{I}_S$

$$0\leftarrow\mathcal{I}_S\leftarrow\begin{array}{c}\mathcal{O}(-4)\\ \oplus\\ 9\mathcal{O}(-5)\\ \oplus\\ \mathcal{O}(-6)\end{array}\nwarrow\begin{array}{c}15\mathcal{O}(-6)\\ \oplus\\ 3\mathcal{O}(-7)\end{array}\longleftarrow\begin{array}{c}7\mathcal{O}(-7)\\ \oplus\\ 3\mathcal{O}(-8)\end{array}\longleftarrow\begin{array}{c}\mathcal{O}(-8)\\ \oplus\\ \mathcal{O}(-9)\end{array}\leftarrow 0$$

Construction
$\mathcal{E} = \mathcal{O}(-1) \oplus \Omega^3(3),\ \mathcal{F} = \ker(5\mathcal{O} \oplus 2\mathcal{O}(1) \xrightarrow{\psi} \mathcal{O}(2))$
where ψ is a special morphism.
see (2.2) for details and (2.4) for a liaison construction,

Cohomology

1				
	1			
		1	3	1
				1

$\mathrm{h}^i(\mathcal{I}_S(p))$

Family properties
irreducible, unirational, of dimension 45
which differs from $\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 44$

2. Degree 11

11.A.

Invariants
$d = 11,\ \pi = 10,\ HK = 7,\ K^2 = -6,\ \chi = 1,\ q = 0,\ N_6 = 10$, [DES]

Classification
$S = S_{min}(p_1, ..., p_6)$ non-minimal Enriques surface
$H = H_{min} - 2E_1 - \sum_2^6 E_i$

Syzygies of $\mathcal{I}_S$

$$0\leftarrow\mathcal{I}_S\leftarrow\begin{array}{c}15\mathcal{O}(-5)\\ \oplus\\ 10\mathcal{O}(-6)\end{array}\longleftarrow\begin{array}{c}3\mathcal{O}(-6)\\ \oplus\\ 26\mathcal{O}(-7)\end{array}\nwarrow 20\mathcal{O}(-8)\longleftarrow 5\mathcal{O}(-9)\leftarrow 0$$

Construction
$\mathcal{E} = 2\Omega^3(3),\ \mathcal{F} = \ker(10\mathcal{O} \xrightarrow{\psi} \mathcal{O}(2))$
where ψ is a special morphism.
see [DES] and (3.7) for details
see also (3.8) for a liaison construction

Cohomology

	2			
		1	5	5

$\mathrm{h}^i(\mathcal{I}_S(p))$

Family properties
irreducible, unirational, of dimension 34
which differs from $\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 36$

11.B1.

Invariants
$d = 11,\ \pi = 11,\ HK = 9,\ K^2 = -11,\ \chi = 1,\ q = 0,\ N_6 = 7$, [DES]

Cohomology

i

	3	1		
			2	1

p

$h^i(\mathcal{I}_S(p))$

Classification
$S = \mathbb{P}^2(p_0, ..., p_{19})$ rational surface
$H = 10l - 4E_0 - 3\sum_1^3 E_i - 2\sum_4^{13} E_j - \sum_{14}^{19} E_k$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}
 & & 10\mathcal{O}(-5) & & 13\mathcal{O}(-6) & & 4\mathcal{O}(-7) & & \\
0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & & \oplus \arrow[ll] & & \oplus \arrow[ll] & & \\
 & & \mathcal{O}(-6) & & 3\mathcal{O}(-7) & & 3\mathcal{O}(-8) & \mathcal{O}(-9) \arrow[luu] & 0 \arrow[l]
\end{tikzcd}$$

Construction
$\mathcal{E} = 3\Omega^3(3),\ \mathcal{F} = \Omega^2(2) \oplus \ker(9\mathcal{O} \xrightarrow{\psi} 2\mathcal{O}(1))$
where ψ is a general morphism.
see (3.23) for details

Family properties
irreducible, unirational
$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 41$

11.B2.

Invariants
$d = 11,\ \pi = 11,\ HK = 9,\ K^2 = -11,\ \chi = 1,\ q = 0,\ N_6 = 7$

Cohomology

i

	3	1		
			2	1

p

$h^i(\mathcal{I}_S(p))$

Classification
$S = \mathbb{P}^2(p_0, ..., p_{19})$ rational surface
$H = 11l - 5E_0 - 3\sum_1^6 E_i - 2\sum_7^{12} E_j - \sum_{13}^{19} E_k$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}
0 & \mathcal{I}_S \arrow[l] & 10\mathcal{O}(-5) \arrow[l] & 12\mathcal{O}(-6) & & 3\mathcal{O}(-7) & & \\
 & & & \oplus \arrow[lu] & & \oplus \arrow[ll] & & \\
 & & & 2\mathcal{O}(-7) & & 3\mathcal{O}(-8) & \mathcal{O}(-9) \arrow[luu] & 0 \arrow[l]
\end{tikzcd}$$

Construction
$\mathcal{E} = 3\Omega^3(3),\ \mathcal{F} = \ker(\Omega^2(2) \oplus 2\Omega^1(1) \xrightarrow{\psi} \mathcal{O})$
where ψ is a general morphism.
see (3.23) for details

Family properties
irreducible, unirational
$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 41$

11.B3.

Invariants
$d = 11,\ \pi = 11,\ HK = 9,\ K^2 = -11,\ \chi = 1,\ q = 0,\ N_6 = 7$

Cohomology

i

	3	1		
			2	1

p

$h^i(\mathcal{I}_S(p))$

Classification
$S = \mathbb{P}^2(p_0, ..., p_{19})$ rational surface
$H = 13l - 5E_0 - 4\sum_1^7 E_i - 2\sum_8^{10} E_j - \sum_{11}^{19} E_k$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}
 & & 10\mathcal{O}(-5) & 14\mathcal{O}(-6) & 6\mathcal{O}(-7) & \mathcal{O}(-8) & \\
0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & \oplus \arrow[l] & \oplus \arrow[l] & \oplus \arrow[l] & 0 \arrow[l] \\
 & & 2\mathcal{O}(-6) & 5\mathcal{O}(-7) & 4\mathcal{O}(-8) & \mathcal{O}(-9) &
\end{tikzcd}$$

Construction
$\mathcal{E} = 3\Omega^3(3),\ \mathcal{F} = \ker(\Omega^2(2) \oplus 2\Omega^1(1) \xrightarrow{\psi} \mathcal{O})$
where ψ is a special morphism.
see (3.23) for details

Family properties
irreducible, unirational
$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 41$

11.C1.

Invariants

$d = 11,\ \pi = 11,\ HK = 9,\ K^2 = -5,\ \chi = 2,\ q = 0,\ N_6 = 4,$ [DES]

Classification

$S = S_{min}(p_1, ..., p_5)$ non-minimal $K3$ surface
$H = H_{min} - 5E_1 - \sum_2^5 E_i$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}
0 & \mathcal{I}_S \arrow[l] & 9\mathcal{O}(-5) \arrow[l] & 8\mathcal{O}(-6) & & & \\
 & & & \oplus \arrow[ul] & & & \\
 & & & 5\mathcal{O}(-7) & 7\mathcal{O}(-8) \arrow[ul] & 2\mathcal{O}(-9) \arrow[l] & 0 \arrow[l]
\end{tikzcd}$$

Construction

$\mathcal{E} = \mathcal{O} \oplus 2\Omega^3(3)$, $\mathcal{F} = \ker(13\mathcal{O} \xrightarrow{\psi} 3\mathcal{O}(1))$
where ψ is a general morphism.
see [DES] or (3.30) for details

Cohomology

i

1				
	2			
			3	2

p

$h^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational
$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 43$

11.C2.

Invariants

$d = 11,\ \pi = 11,\ HK = 9,\ K^2 = -5,\ \chi = 2,\ q = 0,\ N_6 = 4$

Classification

$S = S_{min}(p_1, ..., p_5)$ non-minimal $K3$ surface
$H = H_{min} - 4E_1 - 2E_2 - \sum_3^5 E_i$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}
 & & 9\mathcal{O}(-5) & 9\mathcal{O}(-6) & \mathcal{O}(-7) & & \\
0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & \oplus \arrow[l] & \oplus \arrow[l] & & \\
 & & \mathcal{O}(-6) & 6\mathcal{O}(-7) & 7\mathcal{O}(-8) & 2\mathcal{O}(-9) \arrow[ul] & 0 \arrow[l]
\end{tikzcd}$$

Construction

$\mathcal{E} = \mathcal{O} \oplus 2\Omega^3(3)$, $\mathcal{F} = \ker(13\mathcal{O} \xrightarrow{\psi} 3\mathcal{O}(1))$
where ψ is a special morphism.
see (3.30) for more details

Cohomology

i

1				
	2			
			3	2

p

$h^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational
$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 43$

11.C3.

Invariants

$d = 11,\ \pi = 11,\ HK = 9,\ K^2 = -5,\ \chi = 2,\ q = 0,\ N_6 = 4,$

Classification

$S = S_{min}(p_1, ..., p_5)$ non-minimal $K3$ surface
$H = H_{min} - 3E_1 - 2\sum_2^3 E_i - \sum_4^5 E_j$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}
 & & 9\mathcal{O}(-5) & 10\mathcal{O}(-6) & 2\mathcal{O}(-7) & & \\
0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & \oplus \arrow[l] & \oplus \arrow[l] & & \\
 & & 2\mathcal{O}(-6) & 7\mathcal{O}(-7) & 7\mathcal{O}(-8) & 2\mathcal{O}(-9) \arrow[ul] & 0 \arrow[l]
\end{tikzcd}$$

Construction

$\mathcal{E} = \mathcal{O} \oplus 2\Omega^3(3)$, $\mathcal{F} = \ker(13\mathcal{O} \xrightarrow{\psi} 3\mathcal{O}(1))$
where ψ is a special morphism.
see (3.30) for more details

Cohomology

i

1				
	2			
			3	2

p

$h^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational
$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 43$

11.C4.

Invariants

$d = 11,\ \pi = 11,\ HK = 9,\ K^2 = -5,\ \chi = 2,\ q = 0,\ N_6 = 4,$

Classification

$S = S_{min}(p_1, ..., p_5)$ non-minimal $K3$ surface
$H = H_{min} - 2\sum_1^4 E_i - E_5$

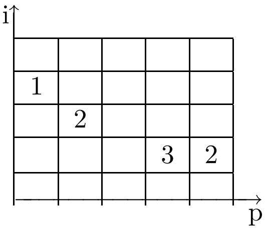
Syzygies of $\mathcal{I}_S$

$$0 \leftarrow \mathcal{I}_S \leftarrow \begin{array}{c} 9\mathcal{O}(-5) \\ \oplus \\ 3\mathcal{O}(-6) \end{array} \longleftarrow \begin{array}{c} 11\mathcal{O}(-6) \\ \oplus \\ 8\mathcal{O}(-7) \end{array} \longleftarrow \begin{array}{c} 3\mathcal{O}(-7) \\ \oplus \\ 7\mathcal{O}(-8) \end{array} \nwarrow 2\mathcal{O}(-9) \leftarrow 0$$

Construction

$\mathcal{E} = \mathcal{O} \oplus 2\Omega^3(3),\ \mathcal{F} = \ker(13\mathcal{O} \xrightarrow{\psi} 3\mathcal{O}(1))$
where ψ is a special morphism.
see (3.30) for more details

Cohomology



1				
	2			
			3	2

$h^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational
$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 43$

11.D.

Invariants

$d = 11,\ \pi = 11,\ HK = 9,\ K^2 = 1,\ \chi = 3,\ q = 0,\ N_6 = 5,$

Classification

$S = S_{min}(p_1)$ non-minimal general type surface
$H = H_{min} - E_1$

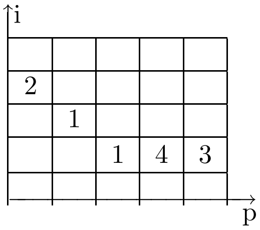
Syzygies of $\mathcal{I}_S$

$$0 \leftarrow \mathcal{I}_S \leftarrow \begin{array}{c} 8\mathcal{O}(-5) \\ \oplus \\ 4\mathcal{O}(-6) \end{array} \longleftarrow \begin{array}{c} 8\mathcal{O}(-6) \\ \oplus \\ 12\mathcal{O}(-7) \end{array} \longleftarrow \begin{array}{c} \mathcal{O}(-7) \\ \oplus \\ 11\mathcal{O}(-8) \end{array} \nwarrow 3\mathcal{O}(-9) \leftarrow 0$$

Construction

$\mathcal{E} = 2\mathcal{O}(-1) \oplus \Omega^3(3),\ \mathcal{F} = \ker(7\mathcal{O} \oplus \mathcal{O}(1) \xrightarrow{\psi} \mathcal{O}(2))$
where ψ is a special morphism.
see (3.32) for more details

Cohomology



2				
	1			
		1	4	3

$h^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational
$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 45$

11.E.

Invariants

$d = 11,\ \pi = 12,\ HK = 11,\ K^2 = -10,\ \chi = 2,\ q = 0,\ N_6 = 9$

Classification

$S = S_{min}(p_1, ..., p_{10})$ non-minimal $K3$ surface
$H = H_{min} - 2E_1 - \sum_2^{10} E_i$

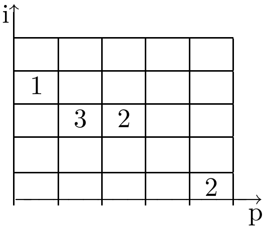
Syzygies of $\mathcal{I}_S$

$$0 \leftarrow \mathcal{I}_S \leftarrow \begin{array}{c} 2\mathcal{O}(-4) \\ \oplus \\ 4\mathcal{O}(-5) \end{array} \nwarrow 7\mathcal{O}(-6) \longleftarrow 2\mathcal{O}(-7) \leftarrow 0$$

Construction

$\mathcal{E} = \mathcal{O}(-1) \oplus 2\Omega^3(3),\ \mathcal{F} = 2\Omega^2(2) \oplus 2\mathcal{O}$
see (3.38) for more details
see (3.39) and (3.41) for liaison

Cohomology



1				
	3	2		
				2

$h^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational, of dimension 48
$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 48$

11.F1.

Invariants

$d = 11,\ \pi = 12,\ HK = 11,\ K^2 = -4,\ \chi = 3,\ q = 0,\ N_6 = 3$

Classification

$S = S_{min}(p_0, ..., p_3)$ non-minimal proper elliptic surface

$H = H_{min} - 2E_0 - \sum_1^3 E_i$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd} 0 & \mathcal{I}_S \arrow[l] & \begin{matrix}\mathcal{O}(-4)\\ \oplus \\ 8\mathcal{O}(-5)\end{matrix} \arrow[l] & 13\mathcal{O}(-6) \arrow[l] & 6\mathcal{O}(-7) \arrow[l] & \mathcal{O}(-8) \arrow[l] & 0 \arrow[l] \end{tikzcd}$$

Construction

$\mathcal{E} = 2\mathcal{O}(-1) \oplus 2\Omega^3(3),\ \mathcal{F} = \Omega^2(2) \oplus \Omega^1(1) \oplus \mathcal{O}$

see (3.43) for more details

see also (3.44) for a liaison construction

Cohomology

i					
	2				
		2	1		
				1	
					1
					p

$h^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational, of dimension 50

which equals $\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 50$

11.F2.

Invariants

$d = 11,\ \pi = 12,\ HK = 11,\ K^2 = -4,\ \chi = 3,\ q = 0,\ N_6 = 3$

Classification

$S = S_{min}(p_1, ..., p_5)$ blown-up, general type Horikawa surface

$H = H_{min} - \sum_1^5 E_i$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd} 0 & \mathcal{I}_S \arrow[l] & \begin{matrix}2\mathcal{O}(-4)\\ \oplus \\ 3\mathcal{O}(-5)\\ \oplus \\ 2\mathcal{O}(-6)\end{matrix} \arrow[l] & \begin{matrix}5\mathcal{O}(-6)\\ \oplus \\ 5\mathcal{O}(-7)\end{matrix} \arrow[l] & \begin{matrix}\mathcal{O}(-7)\\ \oplus \\ 4\mathcal{O}(-8)\end{matrix} \arrow[l] & \mathcal{O}(-9) \arrow[l] & 0 \arrow[l] \end{tikzcd}$$

Construction

$\mathcal{E} = 2\mathcal{O}(-1) \oplus 2\Omega^3(3),\ \mathcal{F} = 2\mathcal{O} \oplus \ker(\Omega^2(2) \oplus \Omega^1(1) \xrightarrow{\psi} \mathcal{O})$

where ψ is a special morphism; see (3.43) for more details

see also (3.43) for a discussion of liaison

Cohomology

i					
	2				
		2	1		
				1	1
					2
					p

$h^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational

$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 50$

11.G1.

Invariants

$d = 11,\ \pi = 12,\ HK = 11,\ K^2 = 2,\ \chi = 4,\ q = 0,\ N_6 = 1$

Classification

$S = S_{min}(p_1)$ non-minimal, general type Horikawa surface

$H = H_{min} - E_1$

Syzygies of $\mathcal{I}_S$

$$0 \leftarrow \mathcal{I}_S \leftarrow 12\mathcal{O}(-5) \longleftarrow 19\mathcal{O}(-6) \longleftarrow 10\mathcal{O}(-7) \longleftarrow 2\mathcal{O}(-8) \leftarrow 0$$

Construction

$\mathcal{E} = 3\mathcal{O}(-1) \oplus \Omega^3(3),\ \mathcal{F} = 2\Omega^1(1)$

see (3.49) for more details

Cohomology

i					
	3				
		1			
				2	
					p

$h^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational

$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 52$

11.G2.

Invariants

$d = 11,\ \pi = 12,\ HK = 11,\ K^2 = 2,\ \chi = 4,\ q = 0,\ N_6 = 1$

Classification

$S = S_{min}$ minimal, general type surface

$H = H_{min}$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}[row sep=tiny]
 & & 2\mathcal{O}(-4) & & & & \\
 & & \oplus & & & & \\
0 & \mathcal{I}_S \arrow[l] & 7\mathcal{O}(-5) \arrow[l] & 10\mathcal{O}(-6) & 3\mathcal{O}(-7) & & \\
 & & \oplus & \oplus \arrow[ul] & \oplus \arrow[l] & & \\
 & & \mathcal{O}(-6) & 3\mathcal{O}(-7) & 3\mathcal{O}(-8) & \mathcal{O}(-9) \arrow[ul] & 0 \arrow[l]
\end{tikzcd}$$

Construction

$\mathcal{E} = 3\mathcal{O}(-1) \oplus \Omega^3(3)$, $\mathcal{F} = \mathcal{O} \oplus \ker(9\mathcal{O} \xrightarrow{\psi} 2\mathcal{O}(1))$

where ψ is a general morphism.

see (3.49) for more details

Cohomology

i↑

3				
	1			
			2	1
				1

→ p

$\mathrm{h}^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational

$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 52$

11.G3.

Invariants

$d = 11,\ \pi = 12,\ HK = 11,\ K^2 = 2,\ \chi = 4,\ q = 0,\ N_6 = 1$

Classification

$S = S_{min}(p_1, p_2)$ non-minimal, general type surface

$H = H_{min} - E_1 - E_2$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}[row sep=tiny]
 & & 2\mathcal{O}(-4) & & & & \\
 & & \oplus & & & & \\
0 & \mathcal{I}_S \arrow[l] & 2\mathcal{O}(-5) \arrow[l] & 3\mathcal{O}(-6) & & & \\
 & & \oplus & \oplus \arrow[ul] & & & \\
 & & 4\mathcal{O}(-6) & 10\mathcal{O}(-7) & 8\mathcal{O}(-8) \arrow[ul] & 2\mathcal{O}(-9) \arrow[l] & 0 \arrow[l]
\end{tikzcd}$$

Construction

$\mathcal{E} = 3\mathcal{O}(-1) \oplus \Omega^3(3)$, $\mathcal{F} = 2\mathcal{O} \oplus \ker(8\mathcal{O} \xrightarrow{\psi} 2\mathcal{O}(1))$

where ψ is a general morphism; see (3.49) for more details

see also (3.50) for a discussion of liaison

Cohomology

i↑

3				
	1			
			2	2
				2

→ p

$\mathrm{h}^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational

$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 52$

11.H.

Invariants

$d = 11,\ \pi = 12,\ HK = 11,\ K^2 = 8,\ \chi = 5,\ q = 0,\ N_6 = 3$

Classification

$S = S_{min}$ minimal, general type surface

$H = H_{min}$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}[row sep=tiny]
 & & \mathcal{O}(-4) & & & & \\
 & & \oplus & & & & \\
0 & \mathcal{I}_S \arrow[l] & 6\mathcal{O}(-5) \arrow[l] & 8\mathcal{O}(-6) & 2\mathcal{O}(-7) & & \\
 & & \oplus & \oplus \arrow[ul] & \oplus \arrow[l] & & \\
 & & 3\mathcal{O}(-6) & 8\mathcal{O}(-7) & 7\mathcal{O}(-8) & 2\mathcal{O}(-9) \arrow[ul] & 0 \arrow[l]
\end{tikzcd}$$

Construction

$\mathcal{E} = 4\mathcal{O}(-1)$, $\mathcal{F} = \ker(4\mathcal{O}(-1) \oplus 2\mathcal{O} \xrightarrow{\psi} \mathcal{O}(1))$

where ψ is a general morphism.

see (3.52) for more details

Cohomology

i↑

4				
		1	3	2
				1

→ p

$\mathrm{h}^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational

$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 54$

11.J.

Invariants

$d = 11,\ \pi = 13,\ HK = 13,\ K^2 = 3,\ \chi = 5,\ q = 0,\ N_6 = 1$

Cohomology

i

4				
	1	1		
				3

p

$h^i(\mathcal{I}_S(p))$

Classification

$S = S_{min}(p_1)$ non-minimal, general type surface

$H = H_{min} - E_1$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd} & & 3\mathcal{O}(-4) & \mathcal{O}(-5) & & \\ 0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & \oplus \arrow[l] & & \\ & & 2\mathcal{O}(-5) & 4\mathcal{O}(-6) & \mathcal{O}(-7) \arrow[ul] & 0 \arrow[l] \end{tikzcd}$$

Construction

$\mathcal{E} = 4\mathcal{O}(-1) \oplus \Omega^3(3)$, $\mathcal{F} = 3\mathcal{O} \oplus \Omega^2(2)$

see (3.56) for more details

see also (3.59) for a liaison construction

Family properties

irreducible, unirational

$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 59$

11.L.

Invariants

$d = 11,\ \pi = 13,\ HK = 13,\ K^2 = 9,\ \chi = 6,\ q = 0,\ N_6 = 0$

Cohomology

i

5				
			1	
				2

p

$h^i(\mathcal{I}_S(p))$

Classification

$S = S_{min}$ minimal, general type surface

$H = H_{min}$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd} & & 2\mathcal{O}(-4) & & & & \\ 0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & & & & \\ & & 5\mathcal{O}(-5) & 10\mathcal{O}(-6) \arrow[ul] & 5\mathcal{O}(-7) \arrow[l] & \mathcal{O}(-8) \arrow[l] & 0 \arrow[l] \end{tikzcd}$$

Construction

$\mathcal{E} = 5\mathcal{O}(-1)$, $\mathcal{F} = 2\mathcal{O} \oplus \Omega^1(1)$

see (3.60) for more details

see also (3.60) for the $(4,4)$ liaison with an elliptic quintic scroll

Family properties

irreducible, unirational

$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 61$

11.M.

Invariants

$d = 11,\ \pi = 14,\ HK = 15,\ K^2 = 10,\ \chi = 7,\ q = 0,\ N_6 = 0$

Cohomology

i

6				
		1		
			1	5

p

$h^i(\mathcal{I}_S(p))$

Classification

$S = S_{min}$ minimal, general type surface

$H = H_{min}$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd} & & \mathcal{O}(-3) & & & \\ 0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & & & \\ & & 4\mathcal{O}(-5) & 5\mathcal{O}(-6) \arrow[ul] & \mathcal{O}(-7) \arrow[l] & 0 \arrow[l] \end{tikzcd}$$

Construction

$\mathcal{E} = 6\mathcal{O}(-1)$, $\mathcal{F} = \Omega^2(2) \oplus \mathcal{O}(1)$

see (3.68) for more details

see also (3.68) for the $(3,5)$ liaison with a Veronese surface

Family properties

irreducible, unirational

$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 68$

11.N.

Invariants

$d = 11,\ \pi = 14,\ HK = 15,\ K^2 = 16,\ \chi = 8,\ q = 0,\ N_6 = 0$

Classification

$S = S_{min}$ minimal, general type surface

$H = H_{min}$

Syzygies of $\mathcal{I}_S$

$$0 \leftarrow \mathcal{I}_S \leftarrow 4\mathcal{O}(-4) \nwarrow \begin{matrix} 2\mathcal{O}(-5) \\ \oplus \\ \mathcal{O}(-6) \end{matrix} \leftarrow 0$$

Construction

$\mathcal{E} = 2\mathcal{O}(-5) \oplus \mathcal{O}(-6),\ \mathcal{F} = 4\mathcal{O}(-4)$

see (3.70) for more details

see also (3.70) for the $(4, 4)$ liaison with a Castelnuovo surface

Cohomology

i

7	1			
				4

p

$\mathrm{h}^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational, of dimension 70

which equals $\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 70$

3. Degree 12

12.A.

Invariants

$d = 12,\ \pi = 13,\ HK = 12,\ K^2 = 0,\ \chi = 3,\ q = 0,\ N_6 = 10$

Classification

$S = S_{min}$ minimal, proper elliptic surface

$H = H_{min}$

Syzygies of $\mathcal{I}_S$

$$0 \leftarrow \mathcal{I}_S \leftarrow \begin{matrix} 3\mathcal{O}(-5) \\ \oplus \\ 12\mathcal{O}(-6) \end{matrix} \nwarrow 30\mathcal{O}(-7) \longleftarrow 21\mathcal{O}(-8) \longleftarrow 5\mathcal{O}(-9) \leftarrow 0$$

Construction

$\mathcal{E} = 2\mathcal{O}(-1) \oplus 2\Omega^3(3),\ \mathcal{F} = \ker(15\mathcal{O} \xrightarrow{\psi} 4\mathcal{O}(1))$

where ψ is a special morphism; see (4.1) for details

see also (4.4) for a $(5, 5)$ liaison with **13.B.**

Cohomology

i

2				
	2			
			4	5

p

$\mathrm{h}^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational, of dimension 49

which differs from $\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 42$

12.B1.

Invariants

$d = 12,\ \pi = 14,\ HK = 14,\ K^2 = -5,\ \chi = 3,\ q = 0,\ N_6 = 4$

Classification

$S = S_{min}(p_1, ..., p_5)$ non-minimal, proper elliptic surface

$H = H_{min} - 2E_1 - \sum_2^5 E_i$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}[column sep=small, row sep=tiny]
0 & \mathcal{I}_S \arrow[l] & 8\mathcal{O}(-5) \arrow[l] & 7\mathcal{O}(-6) & \mathcal{O}(-7) & & \\
 & & & \oplus \arrow[ul] & \oplus \arrow[l] & & \\
 & & & 4\mathcal{O}(-7) & 4\mathcal{O}(-8) & \mathcal{O}(-9) \arrow[ul] & 0 \arrow[l]
\end{tikzcd}$$

Construction

$\mathcal{E} = 2\mathcal{O}(-1) \oplus 3\Omega^3(3)$, $\mathcal{F} = \ker(2\Omega^2(2) \oplus \Omega^1(1) \xrightarrow{\psi} \mathcal{O})$
where ψ is a general morphism.
see (4.7) for details

Cohomology

i

2				
	3	2		
			1	1

p

$\mathrm{h}^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational
$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 47$

12.B2.

Invariants

$d = 12,\ \pi = 14,\ HK = 14,\ K^2 = -5,\ \chi = 3,\ q = 0,\ N_6 = 4$

Classification

$S = S_{min}(p_1, ..., p_5)$ non-minimal, proper elliptic surface

$H = H_{min} - \sum_1^5 E_i$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}[column sep=small, row sep=tiny]
 & & 8\mathcal{O}(-5) & 9\mathcal{O}(-6) & 2\mathcal{O}(-7) & & \\
0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & \oplus \arrow[l] & \oplus \arrow[l] & & \\
 & & 2\mathcal{O}(-6) & 5\mathcal{O}(-7) & 4\mathcal{O}(-8) & \mathcal{O}(-9) \arrow[ul] & 0 \arrow[l]
\end{tikzcd}$$

Construction

$\mathcal{E} = 2\mathcal{O}(-1) \oplus 3\Omega^3(3)$, $\mathcal{F} = \ker(2\Omega^2(2) \oplus \Omega^1(1) \xrightarrow{\psi} \mathcal{O})$
where ψ is a special morphism.
see (4.7) for details

Cohomology

i

2				
	3	2		
			1	1

p

$\mathrm{h}^i(\mathcal{I}_S(p))$

Family properties

irreducible, unirational
$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 47$

4. Degree 13

13.A1.

Invariants

$d = 13,\ \pi = 16,\ HK = 17,\ K^2 = -17,\ \chi = 1,\ q = 0,\ N_6 = 17$, [DES]

Cohomology

i

	6	5	1	
				1

p

$\mathrm{h}^i(\mathcal{I}_S(p))$

Classification

$S = S_{min}(p_1, ..., p_{17})$ non-minimal, Enriques surface

$H = H_{min} - \sum_1^{17} E_i$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}
 & & 5\mathcal{O}(-5) & & & & \\
0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & & & & \\
 & & \mathcal{O}(-6) & 10\mathcal{O}(-7) \arrow[ul] & 6\mathcal{O}(-8) \arrow[l] & \mathcal{O}(-9) \arrow[l] & 0 \arrow[l]
\end{tikzcd}$$

Construction

$\mathcal{E} = 16\mathcal{O}$, $\mathcal{F} = \ker(\mathcal{S}yz_2(N) \xrightarrow{\psi} \mathcal{O})$

where ψ is general and N^*

is a generic module with syzygies:

see (5.3) for more details

$$\begin{tikzcd}
N^* & R(4) \arrow[l] & & & & & & \\
 & & 9R(2) \arrow[ul] & 10R(1) & R & & & \\
 & & & \oplus \arrow[ul] & \oplus \arrow[l] & & & \\
 & & & 16R & 36R(-1) & 25R(-2) \arrow[ul] & 6R(-3) \arrow[l] & 0 \arrow[l]
\end{tikzcd}$$

Family properties

irreducible, unirational

$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 38$

13.A2.

Invariants

$d = 13,\ \pi = 16,\ HK = 17,\ K^2 = -17,\ \chi = 1,\ q = 0,\ N_6 = 17$

Cohomology

i

	6	5	1	
				1

p

$\mathrm{h}^i(\mathcal{I}_S(p))$

Classification

$S = S_{min}(p_1, ..., p_{17})$ non-minimal, Enriques surface

$H = H_{min} - \sum_1^{17} E_i$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}
 & & 5\mathcal{O}(-5) & \mathcal{O}(-6) & & & \\
0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & \oplus \arrow[l] & & & \\
 & & 2\mathcal{O}(-6) & 10\mathcal{O}(-7) & 6\mathcal{O}(-8) \arrow[ul] & \mathcal{O}(-9) \arrow[l] & 0 \arrow[l]
\end{tikzcd}$$

Construction

$\mathcal{E} = 16\mathcal{O}$, $\mathcal{F} = \ker(\mathcal{S}yz_2(N) \xrightarrow{\psi} \mathcal{O})$

where ψ is general and N^*

is a special module with syzygies:

see (5.3) and ff. for more details

$$\begin{tikzcd}
N^* & R(4) \arrow[l] & & & & & & \\
 & & 9R(2) \arrow[ul] & 10R(1) & R & & & \\
 & & & \oplus \arrow[ul] & \oplus \arrow[l] & & & \\
 & & & 16R & 36R(-1) & 25R(-2) \arrow[ul] & 6R(-3) \arrow[l] & 0 \arrow[l]
\end{tikzcd}$$

Family properties

irreducible, unirational

$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 38$

13.B.

Invariants
$d = 13,\ \pi = 16,\ HK = 17,\ K^2 = -11,\ \chi = 2,\ q = 0,\ N_6 = 10$

Cohomology

i

1				
	5	4		
				2

p

$\mathrm{h}^i(\mathcal{I}_S(p))$

Classification
$S = S_{min}(p_1, ..., p_{17})$ non-minimal, $K3$ surface
$H = H_{min} - 7E_0 - \sum_1^{10} E_i$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}[row sep=tiny]
 & & 4\mathcal{O}(-5) & & & & \\
0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & & & & \\
 & & 5\mathcal{O}(-6) & 16\mathcal{O}(-7) \arrow[ul] & 10\mathcal{O}(-8) \arrow[l] & 2\mathcal{O}(-9) \arrow[l] & 0 \arrow[l]
\end{tikzcd}$$

Construction
$\mathcal{E} = \mathcal{O}(-1) \oplus 12\mathcal{O},\ \mathcal{F} = \ker(\mathcal{S}yz_2(N) \xrightarrow{\psi} 2\mathcal{O})$
where ψ is general and N
has syzygies of type:

$$\begin{tikzcd}[row sep=tiny]
N & 5R(3) \arrow[l] & 21R(2) \arrow[l] & 30R(1) & 12R & & & \\
 & & & \oplus \arrow[ul] & \oplus \arrow[l] & & & \\
 & & & 2R & 15R(-1) & 15R(-2) \arrow[ul] & 4R(-3) \arrow[l] & 0 \arrow[l]
\end{tikzcd}$$

see (4.4) for more details
see also (4.4) for the $(5, 5)$ liaison with **12.A.**

Family properties
irreducible, unirational
$\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 40$

5. Degree 14

14.A.

Invariants
$d = 14,\ \pi = 19,\ HK = 22,\ K^2 = -15,\ \chi = 2,\ q = 0,\ N_6 = 22$

Cohomology

i

1				
	7	7	3	

p

$\mathrm{h}^i(\mathcal{I}_S(p))$

Classification
$S = S_{min}(p_1, ..., p_{17})$ non-minimal, $K3$ surface
$H = H_{min} - 4E_0 - \sum_1^4 2E_i - \sum_5^{14} E_i$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}[row sep=tiny]
 & & 4\mathcal{O}(-5) & 2\mathcal{O}(-6) & & \\
0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & \oplus \arrow[l] & & \\
 & & 4\mathcal{O}(-6) & 8\mathcal{O}(-7) & 3\mathcal{O}(-8) \arrow[ul] & 0 \arrow[l]
\end{tikzcd}$$

Construction
$\mathcal{E} = \mathcal{O}(-1) \oplus 15\mathcal{O},\ \mathcal{F} = \mathcal{S}yz_2(N)$
where N^* is a special module with syzygies:

$$\begin{tikzcd}[row sep=tiny]
N & 3R(4) \arrow[l] & 8R(3) & 4R(2) & & & & \\
 & & \oplus \arrow[ul] & \oplus & & & & \\
 & & 2R(2) & 5R(1) \arrow[ul] & & & & \\
 & & & \oplus & & & & \\
 & & & 15R & 38R(-1) \arrow[ul] & 28R(-2) \arrow[l] & 7R(-3) \arrow[l] & 0 \arrow[l]
\end{tikzcd}$$

see (6.1) for more details
see also (6.2) for a liaison construction.

Family properties
irreducible, unirational, of dimension 39
which differs from $\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 38$

6. Degree 15

15.A.

Invariants

$d = 15,\ \pi = 21,\ HK = 25,\ K^2 = -25,\ \chi = 0,\ q = 1,\ N_6 = 25,$ [ADHPR]

Cohomology

i↑

1	10	10	5	

→ p

$\mathrm{h}^i(\mathcal{I}_S(p))$

Classification

$S = S_{min}(p_1, ..., p_{25})$ non-minimal, bielliptic surface

$H = H_{min} - \sum_1^{25} E_i$

Syzygies of $\mathcal{I}_S$

$$0 \leftarrow \mathcal{I}_S \leftarrow \begin{matrix} \mathcal{O}(-5) \\ \oplus \\ 10\mathcal{O}(-6) \end{matrix} \nwarrow 15\mathcal{O}(-7) \longleftarrow 5\mathcal{O}(-8) \leftarrow 0$$

Construction

$\mathcal{E} = 20\mathcal{O},\ \mathcal{F} = \mathcal{S}yz_2(N)$

where N^* is a module with syzygies:

see [ADHPR] for more details

$$N \leftarrow 5R(4) \leftarrow 15R(3) \nwarrow \begin{matrix} 10R(2) \\ \oplus \\ R(1) \\ \oplus \\ 20R \end{matrix} \nwarrow 45R(-1) \leftarrow 30R(-2) \nwarrow \begin{matrix} 5R(-3) \\ \oplus \\ R(-4) \end{matrix} \leftarrow 0$$

Family properties

irreducible, of dimension 25

which equals $\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 25$

15.B1.

Invariants

$d = 15,\ \pi = 21,\ HK = 25,\ K^2 = -25,\ \chi = 0,\ q = 2,\ N_6 = 25$

Cohomology

i↑

1				
2	10	10	5	

→ p

$\mathrm{h}^i(\mathcal{I}_S(p))$

Classification

$S = S_{min}(p_1, ..., p_{25})$ non-minimal, abelian surface

$H = H_{min} - \sum_1^{25} E_i$

Syzygies of $\mathcal{I}_S$

$$0 \leftarrow \mathcal{I}_S \leftarrow \begin{matrix} \mathcal{O}(-5) \\ \oplus \\ 10\mathcal{O}(-6) \end{matrix} \nwarrow 15\mathcal{O}(-7) \longleftarrow 5\mathcal{O}(-8) \leftarrow 0$$

Construction

$\mathcal{E} = 20\mathcal{O},\ \mathcal{F} = \mathcal{S}yz_2(N)$

where N^* is a module with syzygies:

see (7.4) for more details

$$N \leftarrow 5R(4) \leftarrow 15R(3) \nwarrow \begin{matrix} 10R(2) \\ \oplus \\ R(1) \\ \oplus \\ 20R \end{matrix} \nwarrow 45R(-1) \leftarrow 30R(-2) \nwarrow \begin{matrix} 5R(-3) \\ \oplus \\ R(-4) \end{matrix} \leftarrow 0$$

Family properties

irreducible, unirational, of dimension 27

which differs from $\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 25$

15.B2.

Invariants

$d = 15,\ \pi = 21,\ HK = 25,\ K^2 = -25,\ \chi = 0,\ q = 2,\ N_6 = 25,$ [HM],[Au2]

Classification

$S = S_{min}(p_1, ..., p_{25})$ non-minimal, abelian surface

$H = H_{min} - \sum_1^{25} E_i$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}[row sep=tiny] & & 3\mathcal{O}(-5) & & & & \\ 0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & & & & \\ & & 5\mathcal{O}(-6) & 15\mathcal{O}(-7) \arrow[ul] & 10\mathcal{O}(-8) \arrow[l] & 2\mathcal{O}(-9) \arrow[l] & 0 \arrow[l] \end{tikzcd}$$

Cohomology

i ↑

1				
2	10	10	5	

→ p

$h^i(\mathcal{I}_S(p))$

Construction

$\mathcal{E} = \mathcal{O}(-1) \oplus 15\mathcal{O},\ \mathcal{F} = \ker(\mathcal{S}yz_2(N) \xrightarrow{\psi} 2\mathcal{O}(-1))$

where N^* is a module with syzygies:

see [HM] and [DES] for more details

see also [Au2] for the liaison construction.

$$\begin{tikzcd}[row sep=tiny] N & 5R(4) \arrow[l] & 15R(3) \arrow[l] & 10R(2) & & & & \\ & & & \oplus \arrow[ul] & & & & \\ & & & 4R(1) & 2R(1) & & & \\ & & & \oplus & \oplus \arrow[ul] & & & \\ & & & 15R & 35R(-1) & 20R(-2) \arrow[l] & & \\ & & & & & & 2R(-4) \arrow[ul] & 0 \arrow[l] \end{tikzcd}$$

Family properties

irreducible, of dimension 27

which differs from $\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 25$

15.C.

Invariants

$d = 15,\ \pi = 22,\ HK = 27,\ K^2 = -6,\ \chi = 4,\ q = 0,\ N_6 = 14$

Classification

$S = S_{min}(p_1, ..., p_{17})$ non-minimal, general type surface

$H = H_{min} - \sum_0^{8} E_i$

Syzygies of $\mathcal{I}_S$

$$\begin{tikzcd}[row sep=tiny] & & 2\mathcal{O}(-5) & & & \\ 0 & \mathcal{I}_S \arrow[l] & \oplus \arrow[l] & & & \\ & & 7\mathcal{O}(-6) & 12\mathcal{O}(-7) \arrow[ul] & 4\mathcal{O}(-8) \arrow[l] & 0 \arrow[l] \end{tikzcd}$$

Cohomology

↑ i

3				
	7	8	4	

→ p

$h^i(\mathcal{I}_S(p))$

Construction

$\mathcal{E} = 3\mathcal{O}(-1) \oplus 10\mathcal{O},\ \mathcal{F} = \mathcal{S}yz_2(N)$

where N^* is a special module with syzygies:

see $(6, 12)$ for more details

see also $(6, 12)$ for a liaison construction.

$$\begin{tikzcd}[row sep=tiny] N & 4R(4) \arrow[l] & 12R(3) \arrow[l] & 7R(2) & & & & \\ & & & \oplus \arrow[ul] & & & & \\ & & & 5R(1) & & & & \\ & & & \oplus & & & & \\ & & & 10R & 34R(-1) \arrow[ul] & 27R(-2) \arrow[l] & 7R(-3) \arrow[l] & 0 \arrow[l] \end{tikzcd}$$

Family properties

irreducible, of dimension 38

which equals $\chi(\mathcal{N}_{S|\mathbb{P}^4}) = 38$