SKETCH OF SOLUTIONS (HOMEWORK VII)

3.- By Wilson's theorem $18 \equiv 18! \equiv 16!(17)(18) \mod 19$ Therefore $1 \equiv 16!(-2) \mod 19$ i.e. $-10 \equiv 16! \mod 19$ so $9 \equiv 16! \mod 19$

12.- $2^{1000000} \equiv (2^{16})^{62500} \equiv 1^{62500} \equiv 1 \mod 17$

15.- a) $7^{15} \equiv (7^3)^5 \equiv 3^5 \equiv 5 \mod 17$ Therefore $x \equiv 7^{15} \cdot 12 \equiv 5 \cdot 12 \equiv 9 \mod 17$

b) Analogously $4^{17} \equiv 5 \mod 19$ Therefore $x \equiv 5 \cdot 11 \equiv 17 \mod 19$

20.- Notice $168 = 2^3 \cdot 3 \cdot 7$ Since $(42, a) = 1$ we know a is odd, therefore $a^6 \equiv (a^2)^3 \equiv 1 \mod 8$ (since every unit modulo 8 has order 2).Also, by FLT we get $a^6 \equiv (a^2)^3 \equiv 1 \mod 3$ and $a^6 \equiv 1 \mod 7$. These three congruences imply $a^6 \equiv 1 \mod 168$

22.- Since $30 = 2 \cdot 3 \cdot 5$. We only need to solve $n^9 - n \equiv 0 \mod 2, 3, 5$ notice that if $n \equiv 0 \mod 2, 3, 5$ the congruences are satisfied. Otherwise n is a unit $\mod 2, 3, 5$ and we obtain the equivalent set of congruences $n^8 \equiv 1 \mod 2, 3, 5$. These sete of congruences can be simultaneously solved by FLT

23.- Apply FLT to every term of the sum. Adding all the terms we get $p-1 \equiv -1 \mod p$

Section 6.3

2.-

$$17^{45} \equiv 17^{4 \cdot 11} 17 \equiv 1^{11} 17 \equiv 17 \mod 45$$
$$19^{45} \equiv 19^{2 \cdot 22} 19 \equiv 1^{22} 19 \equiv 19 \mod 45$$

7.- We know

$$2^{2^n} \equiv -1 \mod F_n$$

raising each side of the congruence to the power $2^{2^n - n}$ we get

$$\left(2^{2^n}\right)^{2^{2^n - n}} \equiv 2^{2^n \left(2^{2^n - n}\right)} \equiv 2^{2^{2^n}} \equiv 1 \mod F_n$$