SKETCH OF SOLUTIONS (HOMEWORK IV)

4.- The equation we must solve is $19x + 59y = 1706$ with the restriction of x, y being positive. The solution is $x = 37$, $y = 17$

21.- Let x be the number of cocks, y the number of hens and z the number of chickens We have to solve the equations

$$\begin{aligned} x + y + z &= 100 \\ 5x + 3y + \tfrac{z}{3} &= 100 \end{aligned}$$

with the condition of x, y, z being non-negative. The solutions are

$$(x, y, z) = \begin{cases} (0, 25, 75) \\ (4, 18, 78) \\ (8, 11, 81) \\ (12, 4, 84) \end{cases}$$

Section 4.1

5.- Suppose $a = 2n + 1$ then $a^2 = 4n(n + 1) + 1$, since either n or $n + 1$ is even, we have that $8 \mid 4n(n + 1)$ therefore $a^2 \equiv 1 \mod 8$

22.- **Base:** $4 \equiv 1 + 3 \mod 9$

Inductive step: Suppose $4^n \equiv 1 + 3n \mod 9$ then $4^{n+1} \equiv 4 + 12n \equiv 1 + 3 + 12n \equiv 1 + 3(1 + 4n) \mod 9$ therefore we only need to show that $3(1 + 4n) \equiv 3(n + 1) \mod 9$ but $3 + 12n \equiv 3n + 3 \mod 9 \Leftrightarrow 12n - 3n \equiv 0 \mod 9 \Leftrightarrow 9n \equiv 0 \mod 9$

26.- We are looking for solutions of $x(x-1) \equiv 0 \mod p$. Since p is prime, either $p \mid x$ or $p \mid x - 1$ i.e. $x \equiv 0 \mod p$ or $x \equiv 1 \mod p$

28.- a) 42 b) 2 c) $2^{200} = 2^{4*47+12} = (2^{12})(2^4)^{47} \equiv (2^{12})(2^4) \equiv 2^{16} \equiv 18(\mod 47)$

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Section 4.2

2.- a) $x \equiv 3(\mod 7)$ b) $x \equiv 2, 5, 8(\mod 9)$ c) $x \equiv 7(\mod 21)$ d) No solutions e) $x \equiv 812(\mod 1001)$ f) $x \equiv 1596(\mod 1597)$

6.- There are solutions iff $\gcd(12, 30) = 6 \mid c$ therefore iff $c \equiv 0, 6, 12, 18, 24 \mod 30$ in every case there are 6 incongruent solutions $\mod 30$ (By theorem 4.10)

8.- a) 7, b) 9, c) 8, d) 6