SKETCH OF SOLUTIONS (HOMEWORK IV)

4.- The equation we must solve is \(19x + 59y = 1706\) with the restriction of \(x, y\) being positive. The solution is \(x = 37, y = 17\).

21.- Let \(x\) be the number of cocks, \(y\) the number of hens and \(z\) the number of chickens. We have to solve the equations
\[
\begin{align*}
x + y + z &= 100 \\
5x + 3y + \frac{z}{2} &= 100
\end{align*}
\]
with the condition of \(x, y, z\) being non-negative. The solutions are
\[
(x, y, z) = \{(0, 25, 75), (4, 18, 78), (8, 11, 81), (12, 4, 84)\}
\]

Section 4.1

5.- Suppose \(a = 2n + 1\) then \(a^2 = 4n(n + 1) + 1\), since either \(n\) or \(n + 1\) is even, we have that \(8 | 4n(n + 1)\) therefore \(a^2 \equiv 1 \mod 8\).

22.- **Base:** 4 \(\equiv 1 + 3 \mod 9\)

**Inductive step:** Suppose \(4^n \equiv 1 + 3n \mod 9\) then \(4^{n+1} \equiv 4 + 12n \equiv 1 + 3 + 12n \equiv 1 + 3(1 + 4n) \mod 9\) therefore we only need to show that \(3(1 + 4n) \equiv 3(n + 1) \mod 9\) but 3 + 12n \(\equiv 3n + 3 \mod 9\) \(\iff 12n - 3n \equiv 0 \mod 9 \iff 9n \equiv 0 \mod 9\).

26.- We are looking for solutions of \(x(x - 1) \equiv 0 \mod p\). Since \(p\) is prime, either \(p | x\) or \(p | x - 1\) i.e. \(x \equiv 0 \mod p\) or \(x \equiv 1 \mod p\).

28.- a) 42 b) 2 c) \(2^{200} = 2^{47+12} = (2^{12})(2^4)^{47} \equiv (2^{12})(2^4) \equiv 2^{16} \equiv 18 \mod 47\)

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Section 4.2

2.- a) \(x \equiv 3 \mod 7\) b) \(x \equiv 2, 5, 8 \mod 9\) c) \(x \equiv 7 \mod 21\) d) No solutions e) \(x \equiv 812 \mod 1001\) f) \(x \equiv 1596 \mod 1597\)

6.- There are solutions if \(\gcd(12, 30) = 6\) i.e. \(c \equiv 0, 6, 12, 18, 24 \mod 30\) in every case there are 6 incongruent solutions mod 30 (By theorem 4.10)

8.- a) 7, b) 9, c) 8, d) 6