SKETCH OF SOLUTIONS (HOMEWORK III)

5.- a) $\gcd(6, 10, 15) = 1$ b) $\gcd(70, 98, 105) = 7$ c) $\gcd(280, 330, 405, 490) = 5$

7.- a) $1(10) + 1(6) - 1(15) = 1$ b) $0(70) - 1(98) + 1(105) = 7$ c) $8(490) - 17(405) + 9(330) = 5$

19.- Notice that if r is the residue of dividing u by v then $a^r - 1$ is the residue of dividing $a^u - 1$ by $a^v - 1$:

$$a^u - 1 = a^{vq+r} - 1 = (a^v - 1)(a^{v(q-1)} + r + \ldots + a^r) + (a^r - 1)$$

($(a^r - 1)$ is indeed the residue since $r < v \Rightarrow a^r < a^v$)
Therefore we can perform simultaneously the algorithm for finding $\gcd(m, n)$ and $\gcd(a^m - 1, a^n - 1)$ and the result follows.

Section 3.4

4.- a) $2, 5$, b) $2, 3, 5$, c) $2, 3, 5, 7$, d) $3, 5, 7, 11, 13, 23, 29$

10.- Suppose p is a prime in the factorization of a such that $p^t \mid a$ but $p^{t+1} \nmid a$. Let $b = q_1^{s_1} \cdots q_n^{s_n}$ be the prime factorization of b. We know $p^{3t} \mid b^2$ therefore there exists q_i such that $q_i = p$ and $3t + \alpha = 2s_i$ with $\alpha \geq 0$ therefore $2s_i \geq 3t$ i.e. $s_i \geq \frac{3}{2}t > t$ but this implies $p^t \mid b$

16.- We are looking for the exponent of the maximum power of 10 that we can factor out in the product 1000!. Since $10 = 2 \cdot 5$ we are looking for the exponent of the maximum power of 5 that we can factor out from 1000! (this is less than the exponent of the maximum power of 2 that we can factor out since every other number is even). This equals

$$\sum_{j=1}^{4} \left[\frac{1000}{5^j}\right] = 249$$

For finding the number 0's in base 8 we have to find the maximum exponent of a power of 8 that factors out of the product 1000! since $8 = 2^3$ this equals the number of 2's that we can factor out divided by three:

$$\frac{\sum_{j=1}^{9} \left[\frac{100}{2^j}\right]}{3} = 331$$

Section 3.6

2.- a) $x = 1 + 4t$ $y = 1 - 3t$ b) $\gcd(12, 18) = 6 \nmid 50$ therefore there are no solutions c) $x = -121 - 47t$ $y = 77 + 30t$ d) $x = 776 - 19t$ $y = 194 + 5t$ e) $x = 442 - 1001t$ $y = 143t + 102t$

3.- The equation we must solve is

$$122x + 112y = 15286$$

(with the restriction of having positive x and y) this equation has two possible solutions $x = 39$, $y = 94$ and $x = 95$ $y = 33$

6.- The equation we must solve is $18x + 33y = 549$ (in order to solve the number x of oranges and y of grapefruit) with the conditions of x and y being positive and y maximum.The solution is $x = 3$ and $y = 15$ which gives a total of 18 pieces of fruit