

## SKETCH OF SOLUTIONS (HOMEWORK X)

- 3.- DWWDF NDWGD ZQ
- 14.- E is mapped into J and E is mapped into O.  $a = 9$  and  $b = 25$  the message is: WE USE FREQUENCIES OF LETTERS TO DECRYPT SECRET MESSAGES
- 15.-  $C \equiv 17(5P + 13) + 3 \equiv 85P + 224 \equiv 7P + 16 \pmod{26}$

### Section 8.2

- 1.- VSPJXH HIPLKB KIPMIE GTG
- 3.- Look for repeated patterns of letters, the gcd of the lengths of the distances between patterns is likely to be the length of the cipher, or period (say it is  $k$ ). Then perform the frequency-count analysis on characters which are at distance  $k$  from each other.
- 4.- The period is 3. The cipher is BOX. The plaintext is: TOBEO RNOTT OBETH ATIST HEQUE STION WHETH ERTIS NOBLE RINTH EMIND TOSUF FERTH ESLIN GSAND ARROW SOFOU TRAGE OUSFO RTUNE
- 13.-  $C = AP \pmod{26}$  where

$$A = \begin{pmatrix} 11 & 6 \\ 2 & 13 \end{pmatrix}$$

- 18.- DQ BC IG KT AC EX
- 19.-

$$P \equiv \begin{pmatrix} 17 & 4 \\ 1 & 7 \end{pmatrix} C + \begin{pmatrix} 22 \\ 15 \end{pmatrix} \pmod{26}$$

### Section 8.4

- 3.- Since a block of ciphertext  $p$  is less than  $n$ , we must have  $(p, n) = p$  or  $(p, n) = q$ . Therefore the cryptanalyst has a factor of  $n$
- 4.- The probability that it is divisible by  $p$  is  $1/p$  and the probability that it is divisible by  $q$  is  $1/q$ . Also, since 0 is the only integer between 0 and  $n - 1$  which is divisible by both  $p$  and  $q$ , the probability of being divisible by both of them is  $1/pq$ . Using the formula for the probability of the union we get  $P(\gcd(P, n) > 1) = 1/p + 1/q - 1/pq$
- 6.- 101900141066218713492155
- 7.- GR EE TI NG SX
- 11.- Let  $P$  be the plaintext message and the two encrypting exponents  $e_1$  and  $e_2$ . Let  $a = (e_1, e_2)$ . Then there exist integers  $x$  and  $y$  such that  $e_1x + e_2y = a$ . Let  $C_1 \equiv P^{e_1} \pmod{n}$  and  $C_2 \equiv P^{e_2} \pmod{n}$  be the two cipher texts. Since  $C_1, C_2, e_1$  and  $e_2$  are known, and since  $x$  and  $y$  can be computed, we can compute  $C_1^x C_2^y \equiv P^{e_1x} P^{e_2y} \equiv P^{e_1x + e_2y} \equiv P^a \pmod{n}$ . Then computing the  $a$ th roots of  $P^a$  we recover  $P$