SKETCH OF SOLUTIONS (HOMEWORK IV)

1 $20! = 2^{18} \cdot 3^8 \cdot 5^4 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19$

2 In order to find the zeros at the end of the decimal expression for 50! we only need to count how many times are we multiplying times $10 = 5 \cdot 2$ (*why?*). Therefore we only need to count how many times we are multiplying times 5 (because clearly there are more 2's). So let us count: $5, 10, 15, 20, 30, 35, 40, 45$ plus 4 more zeros we will get from $25 = 5^2$ and $50 = 2 \cdot 5^2$. Thus, there are 12 zeros at the end of the decimal expression.

3 Every positive integer n greater than one has at least two positive divisors, namely n and 1. If n only has three factors, then n cannot have two or more prime factors (*why*), therefore it has only one prime factor.That is, $n = p^\alpha$. Again, if n has only 3 factors, then $\alpha = 2$.
Analogously if n has exactly 4 factors then, either $n = pq$ with p and q primes or $n = p^3$

4 Prove the special case when $a = p^\alpha$ with p prime. The general case follows from exercise 7.2 (*why?*)

7.5 a) $5, 9, 13, 17, 21, 29$ b) $21^2 = (21)(21) = (9)(49)$

8.2 a) 9, b) no solutions, c) $1, 3, 5, 7$, d) $3, 4$, e) no solutions

8.3 a) $6, 13$, b) no solutions, c) $5, 18, 31, 44, 57, 70, 83$

8.4 a) no solutions, b) 47 solutions, c) no solutions