SKETCH OF SOLUTIONS (HOMEWORK II)

3.1 a) If $u = du'$ and $v = dv'$ with $d > 1$ then $a = u^2 - v^2 = d(d(u')^2 - d(v')^2)$, $b = d(2du'v')$, and $c = d(d(u')^2 + d(v')^2)$

b) $u = 5$, $v = 3$

d) 1.- u and v have no common factor.

2.- u and v have different parities.

e) Suppose $(a = u^2 - v^2, b = 2uv, c = u^2 + v^2)$ satisfies the conditions in d), let p be a prime which is a common factor of a, b and c, then $p \mid (b + c) = (u + v)^2$ and $p \mid (a + b) = (u - v)^2$ therefore $p \mid (u + v)$ and $p \mid (u - v)$. Therefore $p \mid 2u$ and $p \mid 2v$. Since u and v have different parities, c is odd, therefore p cannot be 2. The only possibility left is that $p \mid u$ and $p \mid v$, but u and v have no common factor. Therefore u and v have no common factor.

3.2 a) $x = \frac{(m-1)^2 - 2}{1+m^2}$, $y = 1 - \frac{2m(m+1)}{1+m^2}$

b) There should be at least one point with rational coordinates.

3.3 $x = \frac{1+m^2}{1-m^2}$, $y = m(\frac{1+m^2}{1-m^2} + 1)$

4.2 a) $(8, 8, 32)$, $(18, 18, 108)$, $(32, 32, 4098)$

b) $(n^2A)^3 + (n^2B)^3 = n^6(A^3 + B^3) = n^6C^2 = n^3c^2$

d) $2a^3 = c^2 \Rightarrow 2 \mid c^2$ therefore $c = 2t$. i.e. $2a^3 = 4t^2$ which implies $a^3 = 2t^2$, but then $2 \mid a$. That is $a = 2s$ and $4s^3 = t^2$, therefore $t = 2v$ and $s^3 = v^2$. Substituting we get that the solution (a, a, c) equals $(2(\sqrt[3]{v})^2, 2(\sqrt[3]{v})^2, 4(\sqrt[3]{v})^3)$ Therefore $(2, 2, 4)$ is the only primitive solution with $a = b$

5.1 $\gcd(12345, 67890) = 15)$, $\gcd(54321, 9876) = 3$