

MAT 126 (Practice Midterm 2) Fall 2003

1. The capital value of an asset such as a machine is sometimes defined as the present value of all future net earnings of the asset. The actual lifetime of the asset may not be known, and since some assets last indefinitely, the capital value may be written in the form

$$\int_0^{\infty} K(t)e^{-rt} dt,$$

where  $K(t)$  is the annual rate of earnings produced by the asset at time  $t$ , and  $r$  is the annual interest rate compounded continuously. Find the capital value of an asset that generates the income rate of \$500 per year, with an interest rate of 10%.

2. Find the following antiderivatives:

a)  $\int \frac{1 + \sin(2x)}{2} dx$

b)  $\int \frac{\ln(1+x)}{1+x} dx$

c)  $\int \ln(1/x) dx$

d)  $\int \frac{x^2 + 5x + 2}{(x+1)(x^2 + 1)} dx$

3. Use the trapezoid rule, the midpoint rule and Simpson's rule to

estimate  $\int_{-0.2}^{0.2} \frac{1}{1+x^4} dx$  using 4 subintervals. Does the

trapezoid rule give an underestimate or overestimate for the exact value of the given definite integral? Justify your answer.

4. Does the following integral converge? Justify your answer!

$$\int_0^{\infty} \frac{\sin^2 x}{(1+x)^2} dx$$

5. Use symmetry to find the area of the region bounded by the curve  $y^2 = x^6(1-x^2)$ . Use the substitution  $x = \sin \theta$  when integrating.