Appendix C - SMSG Axioms for Euclidean Geometry

Everything should be made as simple as possible, but not simpler.
—Albert Einstein (1879–1955)

Introductory Note. The School Mathematics Study Group (SMSG), 1958-1977, developed an axiomatic system designed for use in high school geometry courses, which was published in 1961. The axioms are not *independent* of each other, but the system does satisfy all the requirements for Euclidean geometry; that is, all the theorems in Euclidean geometry can be derived from the system. The lack of independence of the axiomatic system allows high school students to more quickly study a broader range of topics without becoming trapped in detailed study of obvious concepts or difficult proofs. You should compare the similarity and differences between the SMSG axioms and those by Hilbert and Birkhoff. Also, you should compare the SMSG axioms with those found in a high school textbook.

(Some web browsers display some characters incorrectly, an angle shows as ∠.)

Undefined Terms. *point*, *line*, and *plane*

Postulate 1. (*Line Uniqueness*) Given any two distinct points there is exactly one line that contains them.

Postulate 2. (*Distance Postulate*) To every pair of distinct points there corresponds a unique positive number. This number is called the distance between the two points.

Postulate 3. (*Ruler Postulate*) The points of a line can be placed in a correspondence with the real numbers such that:

i. To every point of the line there corresponds exactly one real number.
ii. To every real number there corresponds exactly one point of the line.
iii. The distance between two distinct points is the absolute value of the difference of the corresponding real numbers.

Postulate 4. (*Ruler Placement Postulate*) Given two points P and Q of a line, the coordinate system can be chosen in such a way that the coordinate of P is zero and the coordinate of Q is positive.

Postulate 5. (*Existence of Points*)

a. Every plane contains at least three non-collinear points.
b. Space contains at least four non-coplanar points.

Postulate 6. (*Points on a Line Lie in a Plane*) If two points lie in a plane, then the line containing these points lies in the same plane.

Postulate 7. (*Plane Uniqueness*) Any three points lie in at least one plane, and any three non-collinear points lie in exactly one plane.

Postulate 8. (*Plane Intersection*) If two planes intersect, then that intersection is a line.

Postulate 9. (*Plane Separation Postulate*) Given a line and a plane containing it, the points of the plane that do not lie on the line form two sets such that:

i. each of the sets is convex;
ii. if P is in one set and Q is in the other, then segment $\overline{PQ}$ intersects the line.

Postulate 10. (*Space Separation Postulate*) The points of space that do not lie in a given plane form two sets such that:

i. Each of the sets is convex.
ii. If P is in one set and Q is in the other, then segment $\overline{PQ}$ intersects the plane.

Postulate 11. (*Angle Measurement Postulate*) To every angle there corresponds a real number between 0° and 180°.

Postulate 12. (*Angle Construction Postulate*) Let $\overrightarrow{AB}$ be a ray on the edge of the half-plane H. For every r between 0 and 180, there is exactly one $\overrightarrow{AP}$ with P in H such that $m\angle PAB = r$.

Postulate 13. (*Angle Addition Postulate*) If D is a point in the interior of $\angle BAC$, then $m\angle BAC = m\angle BAD + m\angle DAC$.

Postulate 14. (*Supplement Postulate*) If two angles form a linear pair, then they are supplementary.

Postulate 15. (*SAS Postulate*) Given a one-to-one correspondence between two triangles (or between a triangle and itself). If two sides and the included angle of the first triangle are congruent to the corresponding parts of the second triangle, then the correspondence is a congruence.

Postulate 16. (*Parallel Postulate*) Through a given external point there is at most one line parallel to a given line.

Postulate 17. (*Area of Polygonal Region*) To every polygonal region there corresponds a unique positive real number called the area.

Postulate 18. (*Area of Congruent Triangles*) If two triangles are congruent, then the triangular regions have the same area.

Postulate 19. (*Summation of Areas of Regions*) Suppose that the region R is the union of two regions R_1 and R_2. If R_1 and R_2 intersect at most in a finite number of segments and points, then the area of R is the sum of the areas of R_1 and R_2.

Postulate 20. (*Area of a Rectangle*) The area of a rectangle is the product of the length of its base and the length of its altitude.

Postulate 21. (*Volume of Rectangular Parallelpiped*) The volume of a rectangular parallelpiped is equal to the product of the length of its altitude and the area of its base.

Postulate 22. (*Cavalieri's Principle*) Given two solids and a plane. If for every plane that intersects the solids and is parallel to the given plane the two intersections determine regions that have the same area, then the two solids have the same volume.

School Mathematics Study Group, *Geometry*. New Haven: Yale University Press, 1961.

Mathematics is an interesting intellectual sport but it should not be allowed to stand in the way of obtaining sensible information about physical processes.
—*Richard W. Hamming*, Mathematical Maxims and Minims *(1988)*