MATH 515 Second Midterm

November 12, 2014

Name: ________________________ **ID:** ___________________

Question:	1	2	3	4	Total
Points:	15	15	15	15	60
Score:					

There are 4 problems on 4 pages in this exam (not counting the cover sheet). Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, extra papers, and discussions with friends are not permitted.** If you wish to hold a seance to ask the spirit of Euclid for advice, please do any chanting or other incantations very quietly so as not to disturb other students; please, no candles or incense. A good knowledge of ancient Greek would be very helpful should you do this.

Maybe this time you have enough time to complete this exam. Maybe not. Time will tell.

$\boxed{\text{5 pts.}}$ 1. (a) Give the definition of the **median** of a triangle.

$\boxed{\text{5 pts.}}$ (b) For a circle $\mathcal{C}$ with radius r, define **diameter** in two ways (as a number and as a geometric object).

$\boxed{\text{5 pts.}}$ (c) Let $\mathcal{P}$ be a convex polygon. Give the definition of what it means for a circle $\mathcal{C}$ to be **inscribed** in $\mathcal{P}$.

15 pts. 2. Let $\mathcal{R}$ be a rectangle whose width is greater than its height. What is the geometric locus of all points O which are centers of circles $\mathcal{C}$, where each circle $\mathcal{C}$ is tangent to $\mathcal{R}$ in at least two points?

I meant to add that the circles intersect $\mathcal{R}$ only in points of tangency. If you allow the circles to intersect $\mathcal{R}$ in additional points (as well as the two or more points of tangency), please make this clear. I think this makes the problem harder.

$\boxed{\text{15 pts.}}$ 3. Let congruent circles $\mathcal{C}_1$ and $\mathcal{C}_2$ have centers O and P, respectively. Let ℓ be a secant which is parallel to $\overline{OP}$. Furthermore, suppose ℓ intersects $\mathcal{C}_1$ in points A and B, and intersects $\mathcal{C}_2$ in C and D. Finally, assume points B and C are interior to segment $\overline{AD}$.

Prove that $|AC| = |BD| = |OP|$.

15 pts. 4. Given a circle $\mathcal{C}$ with center O, a point A outside the circle, and a segment $\overline{XY}$, construct a secant to $\mathcal{C}$ passing through A so that the resulting chord is congruent to $\overline{XY}$.

You may assume that you have a movable compass, can draw lines between points, can drop or erect perpendiculars to a given line, and find midpoints of segments. Other constructions need to be specified. **Don't forget to prove your construction works!**