MAT 513 First Midterm

March 8, 2017

Name: _____   **ID:** _____

Question:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:						

There are 5 problems in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** If you use any "alternative facts" as part of your answer for any question, I reserve the right to give you an "alternative grade".

Points will be taken off for writing mathematically false statements, even if the rest of the problem is correct.

You have 90 minutes to complete this exam.

10 points 1. Give the definition of a Cauchy sequence $\{x_n\}$.

$\boxed{\text{10 points}}$ 2. Using the definition of the limit, prove that the sequence $\{x_n\}$ with

$$x_n = \frac{n^2}{n^2 + 4}$$

converges to 1.

3. Suppose that $\{a_n\}$ is a monotone sequence with a subsequence $\{a_{n_j}\}$ that converges. Prove that $\{a_n\}$ also converges.

$\boxed{\text{10 points}}$ 4. Let S and T be nonempty, bounded subsets of $\mathbb{R}$ such that $\sup S \in S$ and $\sup T \in T$.

(a) Let $V = S \cup T$. Prove that V is bounded and $\sup V \in V$.

(b) Let $W = S \cap T$. Is it true that $\sup W \in W$? If so, give a proof. If not, give a counterexample.

10 points | 5. For each of the following infinite series, decide whether it converges or diverges. Justify your answer fully.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n + \sin^2 n}$